To the hydrodynamics model of the microdynamics

Idea

Atoms and molecules are described by Schrödinger equation. The physical assumption, that they are made of a thin matter -- pramatter, attracts the construction of the models, capable to consider this circumstance. We will show that the hydrodynamics model of the micro dynamics from which generalised Schrödinger equation follows is possible.

The initial equations

Let's take the equations of macroscopically model of a viscous liquid. We will apply it to thin matter named the pramatter. We will enter the density ρ of the pramatter and its kinematical viscosity η. Let the magnitude σ characterizes addition dynamic properties of the pramatter. We will express the pramatter behavior by the analogy equations with the hydrodynamics equations of a viscous liquid:

$$\partial_i \left(N^{ij} - \frac{\eta}{\sigma}\Phi^{ij}\right) = \partial_i \Psi^{ij}(1) = F^j$$

Let's give the specified tensors by the expressions:

$$N^{ij} = \rho v^i \otimes v^j = \rho \begin{pmatrix} v^1v^1 & v^1v^2 & v^1v^3 & v^1v^0 \\ v^2v^1 & v^2v^2 & v^2v^3 & v^2v^0 \\ v^3v^1 & v^3v^2 & v^3v^3 & v^3v^0 \\ v^0v^1 & v^0v^2 & v^0v^3 & v^0v^0 \end{pmatrix}, \Phi^{ij} = g^{ik}\varphi_k^j = \begin{pmatrix} \partial_1 f^1 & \partial_2 f^1 & \partial_3 f^1 & \partial_0 f^1 \\ \partial_1 f^2 & \partial_2 f^2 & \partial_3 f^2 & \partial_0 f^2 \\ \partial_1 f^3 & \partial_2 f^3 & \partial_3 f^3 & \partial_0 f^3 \\ \partial_1 f^0 & \partial_2 f^0 & \partial_3 f^0 & \partial_0 f^0 \end{pmatrix}.$$

Here v^i are the components of the four-velocity pramatter, $\delta_{ik}{}^j$ is the Kröneker tensor. We will determine the four-force by the expression

$$F^j = -\Phi \frac{\rho}{\sigma} f^i = -\Phi \frac{\rho}{\sigma} \delta_{ik}{}^j v^i v^k.$$

Let's the four-velocity for the pramatter, leaning on the results received in electrodynamics of moving media. We will choose in physical space-time $T^1 \times R^3$ the coordinates

$$x^1 = x, x^2 = y, x^3 = z, x^0 = ic_g t.$$

We will take some tensors, characterizing the structure of the velocity space by the magnitudes

$$\gamma^{ij} = diag(1,1,1,1), \theta^{ij} = diag(1,1,1,\chi).$$

They have been received earlier in model of electrodynamics without the restriction of the velocity [1]. For the four-dimensional interval and the four-velocity we will obtain

$$d\theta = \frac{ic_g dt}{\sqrt{\chi}}\left(1-\chi\frac{u^2}{c_g{}^2}\right)^{1/2}, v^k = \frac{\sqrt{\chi}}{ic_g}\frac{dx^k}{dt}\left(1-\chi\frac{u^2}{c_g{}^2}\right)^{-1/2}.$$

Microdynamics based on the pramatter

The pramatter in the rest correspond the conditions: $u^1 = u^2 = u^3 = 0$. We have in this case $v^0 = \sqrt{\chi}$. For the velocity tensor, viscous pressure tensor and the force we will receive formulas:

$$N^{ij} = \rho\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & v^0v^0 \end{pmatrix}, \Phi^{ij} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \partial_1(v^0v^0) & \partial_2(v^0v^0) & \partial_3(v^0v^0) & \partial_0(v^0v^0) \end{pmatrix}, F^j = -\frac{\rho}{\sigma}\Phi\begin{pmatrix} 0 \\ 0 \\ 0 \\ \chi \end{pmatrix}.$$

If we use $v^0v^0 = \chi$, we will obtain

$$\partial_i N^{ij} = -i\frac{\rho}{c_g}\frac{\partial\chi}{\partial t} - i\chi\frac{1}{c_g}\frac{\partial\rho}{\partial t},$$

$$\partial_i \Phi^{ij} = \frac{\eta}{\sigma}\left(\nabla^2\chi - \frac{1}{c_g^2}\frac{\partial^2\chi}{\partial t^2}\right) + grad\frac{\eta}{\sigma}\cdot grad\chi - \frac{1}{c_g^2}\frac{\partial}{\partial t}\left(\frac{\eta}{\sigma}\right)\cdot\frac{\partial\chi}{\partial t},$$

$$F^j = -\Phi\frac{\rho}{\sigma}\chi.$$

Let's give the designations:

$$\bar{h}_1(l) = \frac{\sigma}{c_g}, \eta = 0{,}5\bar{h}_2{}^2(l).$$

The magnitudes $\bar{h}_j(l), j = 1{,}2$ characterize empirical properties of the pramatter. They should get out according to experiment and can be subordinated to the dynamic equations or some other restrictions.

The fourth component of the pramatter velocity is described by the equation

$$i\bar{h}_1(l)\frac{\partial\chi}{dt} = -\frac{\bar{h}_2^{\,2}(l)}{2\rho}\nabla^2\chi + \Phi(l)\chi + \Pi_1,$$

$$\Pi_1 = \frac{1}{c_g^2}\frac{\eta}{\sigma}\frac{\partial^2\chi}{\partial t^2} - \frac{\sigma}{\rho}grad\frac{\eta}{\sigma}\cdot grad\chi + \frac{\sigma}{\rho}\frac{1}{c_g^2}\frac{\partial}{\partial t}\left(\frac{\eta}{\sigma}\right)\frac{\partial\chi}{\partial t} - i\frac{\partial\ln\rho}{\partial t}\frac{\sigma}{c_g}\chi.$$

We will obtain the Schrödinger equation if we will make some replacements:

fourth component of the velocity χ on wave function ψ,

the magnitude $\bar{h}_1(l)$ on the Plank constanta $\hbar$,

variable pramatter density ρ on the constant mass m of one particle,

the potential Φ on the potential V.

Besides, it is necessary to accept conditions:

the equality of the pair various and generally variable empirical magnitudes to a constant of Planck in the form of $\bar{h}_1(l) = \bar{h}_2(l) = \hbar,$,

$\Pi_1 = 0$, these condition limits a range of dynamic change of model magnitude.