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DYNAMIC NATURE OF THE RELATIVISTIC EFFECTS IN ELECTRODYNAMICS

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New method proposed for the description of the relativistic effects in classical electrodynamics. It based on the generalization of the material equitation, which depends on new physical magnitude, named the rate of the relation. In this approach we can describe all relativistic effects in electrodynamics dynamically without the velocity restriction and singularities. The dynamical influence of the classical equipment at the electromagnetic field is described. Four canonical 0-cogomoloqically dependent metrics it is induced in physical theories.

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Part 1
Generalization of Maxwell's electrodynamics in moving media

RESUME. *Generalization of Maxwell's electrodynamics in moving media is suggested, which, first, does not resort to the Einstein special relativity theory; second ,bases its calculations and experiments on Newton's space; third ,naturally incorporates superlight velocities and indicates the requirements for the latter to be discovered, and fourth ,describes the classical experiments of Bradley, Michelson, Fizeau, and Doppler in a unified manner.*

Introduction

We shall show the possibility of DYNAMIC description of a change of the inertial factors for an electromagnetic field within the framework of the NEWTONIAN space - time, in a single coordinate system, when the frame of reference is considered as a physical environment capable of influencing the parameters of the field.

1.1. Maxwell's dynamic equations in the Newtonian space-time

We will start with the concept of a single observer who has the standard of length and time according to Newton's space-time model $R^3 \times T^1$. The physical laws of Maxwell's electrodynamics in $R^3 \times T^1$ can be determined in terms of the three-dimensional operators $\nabla \times$ and $\nabla \cdot$ and they have a vector form:

$$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}, \qquad \nabla \cdot \vec{B} = \vec{0},$$

$$\nabla \cdot \vec{D} = 4\pi\rho, \qquad \nabla \times \vec{H} = \frac{1}{c}\frac{\partial \vec{D}}{\partial t} + 4\pi\frac{\vec{J}}{c}.$$

In the algebra $F(4)$ elements form

$$F_{mn} = \begin{pmatrix} 0 & B_z & -B_y & -iE_x \\ -B_z & 0 & B_x & -iE_y \\ B_y & -B_x & 0 & -iE_z \\ iE_x & iE_y & iE_z & 0 \end{pmatrix},$$

$$H^{ik} = \begin{pmatrix} 0 & H_z & -H_y & -iD_x \\ -H_z & 0 & H_x & -iD_y \\ H_y & -H_x & 0 & -iD_z \\ iD_x & iD_y & iD_z & 0 \end{pmatrix},$$

Maxwell's equations acquire the tensor form:

$\partial_{[k}F_{mn]} = 0$, $\partial_k H^{ik} = S^i$,

where ∂_k is the covector of partial derivatives, for example over the coordinates

$x^1 = x$, $x^2 = y$, $x^3 = z$, $x^0 = ict$.

Physically speaking, these sets of equations are equivalent; however, it is more convenient to carry out the mathematical analysis of general problems in the tensor form.
Starting from these equation, and not resorting to the concept of an ether, we will describe in a unified manner the experiments of Bradley [1], Fizeau [2], Michelson [3], and Doppler [4], the "constancy" of the speed of light in vacuum [5], following the model of dynamic change of field parameters in the NEWTONIAN space-time.

1.2. Generalized connections between fields and inductions in Maxwell's electrodynamics

For an isotropic medium at rest the connection between fields and inductions has the form: $\vec{D} = \varepsilon\vec{E}$, $\vec{B} = \mu\vec{H}$, where ε and μ are the dielectric and magnetic permeability's.
In the version considered by Minkowski [6], the medium is a secondary radiation source, so the medium velocity $\vec{U}_m$ is identical with this velocity of the radiation source:

$$\vec{D} + \left[\frac{\vec{U}_m}{c} \times \vec{H}\right] = \varepsilon\left(\vec{E} + \left[\frac{\vec{U}_m}{c} \times \vec{B}\right]\right),$$

$$\vec{B} + \left[\vec{E} \times \frac{\vec{U}_m}{c}\right] = \mu\left(\vec{H} + \left[\vec{D} \times \frac{\vec{U}_m}{c}\right]\right).$$

We will seek connections between the fields F_{mn} and inductions H^{ik} [7] in the form:

$$H^{ik} = \Omega^{im}\Omega^{kn}F_{mn} .$$

Let Ω^{im} be equal to

$$\Omega^{im} = \alpha\left(\Theta^{im} + \beta U^i U^m\right),$$

where α and β are scalar functions, $\Theta^{im} = diag\,(1, 1, 1, \chi)$ is the metric tensor in $R^3 \times T^1$, and $\chi = \det\Theta^{im}$, $U^i = dx^i / d\Theta$ represents the four-velocities, without invoking SRT.
Here we have $d\Theta^2 = \Theta_{ij}dx^i dx^j$, and the inverse tensor can be specified in two ways:
a) $\Theta_{ij}\Theta^{jk} = \delta_i^k$, b) $\Theta_{ij} = b_{ik}b_{jl}\Theta^{kl}$, where b_{ij} are additional tensors.
In such a statement the expression for Ω^{im} has been found 8] by solving a system of nonlinear algebraic equations following from the generalized formal connection for fields and inductions, when the connections are considered for the velocity equal to zero. Then

$$\Omega^{im} = \frac{1}{\sqrt{\mu}}\left[\Theta^{im} + \left(\frac{\varepsilon\mu}{\chi} - 1\right)U^i U^m\right].$$

The tensor Ω^{im} has no singularity at $\chi = 0$. Really,

$$d\Theta = \frac{icdt}{\sqrt{\chi}}\left(1 - \chi\frac{U^2}{c^2}\right)^{1/2}, \quad U^k = \frac{dx^k}{d\Theta} = \frac{\sqrt{\chi}}{ic}\frac{dx^k}{dt}\left(1 - \chi\frac{U^2}{c^2}\right)^{-1/2} .$$

For the velocities $U_n = \Theta_{nk} U^k$ we have $U^k U_k = 1$.In view of the antisymmetry of F_{mn} and H^{ik}, we have

$H^{ik} = \Omega^{ikmn} F_{mn}, \Omega^{ikmn} = 0{,}5\left(\Omega^{im}\Omega^{kn} - \Omega^{in}\Omega^{km}\right)$
with the conditions
$\Omega^{ikmn} = -\Omega^{iknm} = -\Omega^{kimn}$.
Maxwell's generalized equations take the vector form [9]:

$$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}, \quad \nabla \cdot \vec{B} = \vec{0}, \nabla \cdot \vec{D} = 4\pi\rho, \qquad \nabla \times \vec{H} = \frac{1}{c}\frac{\partial \vec{D}}{\partial t} + \frac{4\pi}{c}\vec{j},$$
$$\vec{D} + \chi\left[\frac{\vec{U}}{c} \times \vec{H}\right] = \varepsilon\left(\vec{E} + \left[\frac{\vec{U}}{c} \times \vec{B}\right]\right), \qquad \vec{B} + \chi\left[\vec{E} \times \frac{\vec{U}}{c}\right] = \mu\left(\vec{H} + \left[\vec{D} \times \frac{\vec{U}}{c}\right]\right).$$

1.3. Main model problem

Let a radiation source move around the Earth in vacuum with instantaneous velocity $\vec{U}_{fs}$, which is the velocity of the primary radiation source $\vec{U}\Big|_{\rho=0} = \vec{U}_{fs}$ .Let the radiation spread from empty space into the atmosphere of Earth's ,which has density ρ, in which for $\rho = \rho_0$ the velocity of the secondary radiation source is equal to the velocity of the physical medium U_m:
$\vec{U}\Big|_{\rho=\rho_0} = \vec{U}_m$.

Let us introduce the velocity $\vec{U} = \vec{U}\left(\vec{U}_{fs}, \vec{U}_m, w(\rho)\right)$ assuming that it also depends on the functional $w(\rho)$, which is named the phase of the inertia of the electromagnetic field. We will assume that in agreement with the indicated physical formulation [7], the velocity $\vec{U}$ is governed by the relaxation equation
$$\frac{d\vec{U}}{d\xi} = -P_0\left(\vec{U} - \vec{U}_m\right), \ \vec{U}\Big|_{\xi=0} = \vec{U}_{fs} .$$
Here P_0 is the relaxation constant, $\xi = \dfrac{\rho}{\rho_0}$. The solution of the relaxation equation is

$$\vec{U} = (1-w)\vec{U}_{fs} + w\vec{U}_m, \quad w = 1 - \exp\left(-P_0\frac{\rho}{\rho_0}\right).$$
We have the conditions

$$\vec{U}\Big|_{\rho=0_0} = \vec{U}_{fs}, \qquad w\Big|_{\rho=0} = 0, \qquad \vec{U}\Big|_{\rho=\rho_0} = \vec{U}_m, \quad w\Big|_{\rho=\rho_0} = 1.$$

We require that $\chi = w$. The solution of the indicated problem is then in general possible.

1.4. Solution of Maxwell's generalized equations with $w = const$

When $w = const$,the equations for the field potentials A_m in their four-dimensional form are [10]:

$$\left[\Theta^{kn}\frac{\partial}{\partial x^k}\frac{\partial}{\partial x^n}-(\varepsilon\mu-w)\left(U^k\frac{\partial}{\partial x^k}\right)^2\right]A_m=-\mu U^i\Theta_{im}$$

with the calibration condition:

$$\Theta^{kn}\frac{\partial A_n}{\partial x^k}-(\varepsilon\mu-w)\frac{\partial A_l}{\partial x^k}U^lU^k=0.$$

For vector $\vec{A}$ and scalar φ potentials, according to their definitions

$$\vec{E}=-\frac{1}{c}\frac{\partial\vec{A}}{\partial t}-\nabla\varphi,\qquad \vec{B}=\nabla\times\vec{A}$$

we obtain

$$L\vec{A}=-\frac{4\pi\mu}{c}\left\{\vec{J}+\frac{\sigma\Gamma^2}{\sigma+w}\frac{\vec{U}}{c}\left(w\vec{U}\vec{J}-c^2\rho\right)\right\},L\varphi=-4\pi\mu\frac{\Gamma^2}{w+\sigma}\left\{\rho\left(1-\varepsilon\mu\frac{U^2}{c^2}\right)+\sigma\frac{\vec{U}\vec{J}}{c^2}\right\},$$

and the calibration condition

$$\left(\nabla\cdot\vec{A}+\frac{w}{c}\frac{\partial^2}{\partial t^2}\right)-\frac{\sigma\Gamma^2}{c^2}\left(\frac{\partial}{\partial t}+\vec{U}\nabla\right)\left(\vec{U}\vec{A}-c\varphi\right)=0,$$

where

$$L=\left(\Delta-\frac{w}{c^2}\frac{\partial^2}{\partial t^2}\right)-\sigma\frac{\Gamma^2}{c^2}\left(\frac{\partial}{\partial t}+\vec{U}\nabla\right)^2,\quad \sigma=\varepsilon\mu-w,\quad \Gamma^2=\left(1-w\beta^2\right)^{-1},\quad \beta=\frac{U}{c}.$$

The Green function for the vector equations is indicated in [7]:

$$G_0(\vec{r},t)=16\pi^4\mu\left(r^2+\xi^2\right)^{-1/2}\delta\left(t-\frac{1}{c}\frac{\varepsilon\mu-\beta^2w^2}{\left(1-w\beta^2\right)\sqrt{\varepsilon\mu}}\left(r^2+\xi^2\right)^{1/2}\right).$$

It is given in a cylindrical coordinate system, the position vector for which has length $R=\left(\rho^2+z^2\right)^{1/2}$, and the values are equal to

$$r^2=\rho^2\frac{\varepsilon\mu\left(1-w\beta^2\right)}{\varepsilon\mu-\beta^2w^2},\ \xi=z-\frac{\varepsilon\mu-w}{\varepsilon\mu-\beta^2w^2}Ut.$$

When $\beta=0$, we have the Green function for the medium at rest without dispersion:

$$G_0(\vec{r},t)\Big|_{\vec{U}=0}=16\pi^4\mu\frac{1}{R}\sigma\left(t-\frac{R\sqrt{\varepsilon\mu}}{c}\right).$$

The Green function differs from zero on the surface:

$$t=\frac{1}{c}\frac{\varepsilon\mu-\beta^2w^2}{\left(1-w\beta^2\right)\sqrt{\varepsilon\mu}}\left(\rho^2\frac{\varepsilon\mu\left(1-w\beta^2\right)}{\varepsilon\mu-\beta^2w^2}+\left(z-\frac{\varepsilon\mu-w}{\varepsilon\mu-\beta^2w^2}Ut\right)^2\right)^{1/2}.$$

This is an ellipsoid of rotation whose symmetry axis coincides with $\vec{U}$, and the position of the center is given by

$$z_0=Ut\frac{\varepsilon\mu-w}{\varepsilon\mu-\beta^2w^2}.$$

The center of ellipsoid moves with the velocity

$$U_0 = U\frac{\varepsilon\mu - w}{\varepsilon\mu - \beta^2 w^2}.$$

The semi axes of an ellipsoid are equal to

$$a = ct\left(\frac{1 - w\beta^2}{\varepsilon\mu - \beta^2 w^2}\right)^{1/2}, \quad b = ct\frac{\sqrt{\varepsilon\mu}\,(1 - w\beta^2)}{\varepsilon\mu - \beta^2 w^2}.$$

The dispersion equation for the electromagnetic field has standard form [11]:

$$c^2K^2 = w\omega^2 + \Gamma^2(\varepsilon\mu - w)\left(\omega - \vec{K}\cdot\vec{U}\right)^2, \Gamma^2 = \left(1 - w\beta^2\right)^{-1},$$

where $\vec{K}$ is the wave vector. This yields the expression for the group velocity:

$$\vec{V}_g = \frac{\partial\omega}{\partial\vec{K}} = c\frac{\vec{K} + \sigma\Gamma^2 c^{-2} U\left(\omega - \vec{K}\cdot\vec{U}\right)}{\frac{\omega}{c} w + \sigma\Gamma^2 c^{-1}\left(\omega - \vec{K}\cdot\vec{U}\right)}.$$

In a nonrelativistic limit

$$\vec{V}_g = \frac{c}{n}\frac{\vec{K}}{K} + \left(1 - \frac{w}{n^2}\right)\left[(1 - w)\vec{U}_{fs} + w\vec{U}_m\right].$$

1.5. Analysis of the expressions obtained

1. At $w = 0$ we have

$$\vec{V}_g = c\frac{\vec{K}}{K} + \vec{U}_{fs}.$$

Thus, in the generalized model of electromagnetic events the field moves such that the center of the surface on which the Green function is nonzero moves with the velocity $\vec{U}_{fs}$, and the semi axes of the ellipse in this case are equal, giving a sphere.. This picture corresponds to an intuitive comprehension of the fact, according [12], in the absence of external influences, the field in vacuum retains its inertia.

2. The generalized electrodynamics of Maxwell's one is consistent with the experiments of Michelson [3]. According to the conditions of his experiment, the velocity of the medium was equal to zero, $\vec{U}_m = 0$, just as the velocity of the radiation source. For this reason we have the radiation velocity to be independent of the direction:

$$\vec{V}_g = \frac{c}{n}\frac{\vec{K}}{K}.$$

3. The generalized electrodynamics of Maxwell is consistent with the experiment of Fizeau [2]. According to the experimental conditions $\vec{U}_{fs} = 0$ and $w = 1$, therefore the velocity is equal to

$$\vec{V}_g = \frac{c}{n}\frac{\vec{K}}{K} + \left(1 - \frac{1}{n^2}\right)\vec{U}_m.$$

Conclusions

The generalization of Maxwell's electrodynamics, which allows one to describe in a unified manner a vast quantity of experimental data without resorting to the special relativity theory, is possible if the connections between the fields and inductions is taken into account.

The functional $w(\rho)$ and also the velocity that specifies the external inertia of the field $\vec{U} = (1 - w)\vec{U}_{fs} + w\vec{U}_m$ change in this case dynamically, which are the determining factors for the velocity $\vec{V}_g$ and the frequency ω of the electromagnetic field.

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PART 2
Dynamic mechanism of the external velocity transformations into a proper frequency of an electromagnetic field

RESUME. *The earlier unknown dynamic mechanism of the transformation of the velocity that specifies the external inertia of a field, into a proper frequency of an electromagnetic field is found. It is shown that a particle of nonzero rest mass can be limited at the particle velocity equal the light speed in vacuum.*

Introduction

Part 1 of this article suggests a generalization of Maxwell's electrodynamics in which the dynamic equations are used without involving any new elements, while the connections between fields and inductions are extended. The generalized connections contain the velocity of a primary radiation source $\vec{U}_{fs}$, the medium velocity $\vec{U}_m$, and also new quantity, namely, the external inertia phase of the electromagnetic field $w(\rho) = 1 - \exp\left(-P_0 \rho / \rho_0\right)$, where ρ - is the atmosphere density.

The calculation of the field parameters and analysis of experimental data are carried out in Newton's space model. The absolute character of length and time are the foundation of the proposed algorithm for a dynamic change in the inertial parameters of the field.

The equations for field potentials, following from Maxwell's generalized equations, are obtained. The Green's function is found and its physical consequences are analyzed. A generalized expression for the field group velocity is obtained. The dependence of the field velocity in vacuum on the primary radiation source velocity is shown.

Now, we will study a dynamics of the field frequency.

2.1. New requirement on a wave phase

The group velocity of an electromagnetic field for $w \to 1$ does not depend on $\vec{U}_{fs}$. Physically this change of the speed can and must be exhibited as a change in frequency. Since a dynamic change in the speed is considered, consequently, there will also be dynamic change in the frequency ω. To understand, the manner in which this occurs, we supplement the dispersion equation with the generalized phase requirement [1]:

$$\frac{\omega - \vec{K} \cdot \vec{U}_\xi}{\left(1 - w_\xi \frac{U_\xi^2}{c^2}\right)^{1/2}} = const\,.$$

This requirement does not follow directly from Maxwell's equations and, consequently, we will assume that the velocity $\vec{U}_\xi$ can be different from the radiation carrier velocity $\vec{U}$. By analogy with the already adopted algorithm and the model of the analysis, we will consider the new velocity $\vec{U}_\xi$ in the following form:

$$\vec{U}_\xi\left(\vec{U}_{fs}, \vec{U}_m, w_\xi(\rho)\right) \neq \vec{U},$$

assigning for it the equation of the relaxation type [2]:

$$\frac{d\vec{U}_\xi}{d\xi} = -P_\xi\left(\vec{U}_\xi - \vec{U}_*\right), \; \vec{U}_\xi\Big|_{\xi=0} = \vec{U}_{fs}.$$

In order to preserve $\vec{U}_{fs}$ as a function on of $\vec{U}_\xi$, we use as the relaxation value

$$\vec{U}_* = \vec{U}_{fs} + \vec{U}_m,$$

which is permissible in Newton's model.We have the solution

$$\vec{U}_\xi = \vec{U}_{fs} + w_\xi \vec{U}_m, \; w_\xi = 1 - \exp\left(-P_\xi \frac{\rho}{\rho_0}\right).$$

The situation appears thus: from the kinematic point of view, because of the interaction with the medium, the velocity $\vec{U}_{fs}$, disappears and it is not exhibited in the group velocity; from the energy point of view, it is transformed into the frequency ω. This can be achieved because the role and functions of the dispersion and phase requirements, are complementary.

2.2. Dynamics of the Doppler effect and aberrations in Maxwell's electrodynamics

We will adopt the point of view that the change of the parameters of an electromagnetic field happens only because of its interaction with the medium or with outer fields. Let us consider how these processes occur in the generalized electromagnetic model. Let us analyze the model problem:

The radiation with an initial frequency ω_0 and wave vector $\vec{K}_0$ from a radiation source moving in vacuum with the velocity $\vec{U}_{fs}$ is spread to the Earth surface, on which there is an observer.

The atmosphere is at rest, $\vec{U}_m = 0$. It is required to calculate the manner in which the frequency ω and wave vector $\vec{K}$ change because of the interaction of radiation with the medium. Let $w = w_\xi$ .Using the equations obtained, we will unite in a uniform system the dispersion and phase requirements [3]:

$$c^2K^2 - w\omega^2 = \Gamma^2(\varepsilon\mu - w)\left(\omega - \vec{K}\cdot\vec{U}\right)^2,$$

$$\omega = \omega_0\left(1 - wU_\xi^2/c^2\right)^{1/2} + \vec{K}\cdot\vec{U}_\xi,$$

$$\Gamma^2 = \left(1 - wU^2/c^2\right)^{-1}.$$

We assume that $K_{y_0} = 0$, $K_z = K_{z_0}$ .We find the dependence of ω, K_x on the initial values of ω_0, K_{z_0} .We transform, accurate to $\left(U_{fs}/c\right)^2$, the dispersion equation to the form

$$AK_x^2 + BK_x + P = 0.$$

The coefficients are

$$A = 1 - a\frac{U_{fs}^2}{c^2}, \qquad a = w + \varepsilon\mu w^2 - w^3,$$

$$B = w\frac{w_0}{c}\frac{U_{fs}}{c}b, \; b = 1 + \varepsilon\mu - w,$$

$$P = \frac{w_0^2}{c^2}\frac{U_{fs}^2}{c^2}q, \; q = w^2 - 2w^3 + w^4 + 2\varepsilon\mu w^2 - w^3\varepsilon\mu.$$

We calculate a, b, q for $\varepsilon\mu = 1$.Analysis has shown that the solution can be expressed by the function

$$\Phi = w\left[(2 - w) + (1 - w)^{1/2}\right].$$

We have for K_x a nonlinear dependence on w

$$K_x = \Phi\frac{\omega_0}{c}\frac{U_{fs}}{c}.$$

The aberration angle is defined by the expression

$$\tan\alpha = \frac{K_x}{K_z} = \frac{U_{fs}}{c}\Phi.$$

The connection of initial and intermediate frequencies is given by dependence

$$\omega = \omega_0\left[\left(1 - w\frac{U_{fs}^2}{c^2}\right)^{1/2} + \Phi\frac{U_{fs}^2}{c^2}\right].$$

According to the calculations, far from the Earth surface we have

$$K_x = 0, \; K_z = -\frac{\omega_0}{c}, \; \omega = \omega_0.$$

As the Earth is approached, ω and K_x vary continuously because of the change in w. For $w = 1$ we obtain

$$K_x = \frac{\omega_0}{c}\frac{U_{fs}}{c}, \; \omega = \omega_0\left(1 - \frac{U_{fs}^2}{c^2}\right)^{-1/2}.$$

These values agree with Bradly's experiment and with the formula for the Doppler cross effect. The same results are obtained by the methods of the special relativity theory.

The special relativity theory, as is typical of a kinematic theory, connects initial and final parameters of the field. It is possible to consider the special theory of relativity as corresponding to «black box», given the input parameters, the values at the output of the box are prescribed, but the process itself is not analyzed. The generalized model indicates the laws of the dynamics of the processes. We have

$$\omega = \omega_0 + \left(\Phi - \frac{1}{2}w\right)\frac{U_{fs}}{c}\omega_B,$$

$$\vec{V}_g \equiv \frac{c}{n}\frac{\vec{K}}{K} + \left(1 - \frac{w}{n^2}\right)(1 - w)\vec{U}_{fs}$$

where $\omega_B = \omega_0 \frac{U_{fs}}{c}$.

2.3. New effects in the generalized Maxwell's electrodynamics

2.3.1. Unlimited velocities of an electromagnetic field in vacuum.

In vacuum we have $\rho = 0$ and, consequently, $w = 0$. The field group velocity

$$\vec{V}_g = c\frac{\vec{K}}{K} + \vec{U}_{fs}$$

depends on the velocity of the initial radiation source. The wave front surface represents a sphere, because $a = b = c_0 t$ and its centre moves with the velocity

$$\vec{U}_* = \vec{U}_{fs}.$$

This is the pattern in which the radiation propagates in the new model. It corresponds to the idea suggested by Ritz [4]. Because of the interaction with the medium, in particular with the frame of reference, the velocity $\vec{U}_{fs}$ can vanish. Precisely this happens in all of the schemes for direct measurement of the speed of light in vacuum [5]. Therefore it is possible to consider that the generalized model of electromagnetic phenomena agrees with the "constancy" of light speed in vacuum, demonstrating that for finding the dependence, only indirect experiments are suitable, when measurement without the influence on the quantity $\vec{U}_{fs}$, is possible.

If the radiation moves in a gravitational field, its influence on the inertia of the radiation carrier is possible. This note can turn out to be important for the analysis of radiation transfer in outer space.

2.3.2. Superlight velocities in a moving rarefied gas.

Let the radiation source be at rest with respect to the observer $\vec{U}_{fs} = 0$, and the medium - gas stream moves with the velocity $\vec{U}_m$. Then the group velocity of the field is

$$\vec{V}_g = \frac{c}{n}\frac{\vec{K}}{K} + \left(1 - \frac{w}{n^2}\right) w \vec{U}_m.$$

For index of refraction close to unity, the value $w = 0{,}5$ will maximize the correction term. The velocity will then be

$$\vec{V}_g^{\max} = c_0 \frac{\vec{K}}{K} + \frac{1}{4}\vec{U}_m$$

In the special relativity theory we have different results. The group field velocity depends on the Fresnel classical coefficient according the formula

$$\vec{V}_g = \frac{c}{n}\frac{\vec{K}}{K} + \left(1 - \frac{1}{n^2}\right)\vec{U}_m.$$

Since $n = 1 + Q_\lambda$, where $Q_\lambda \cong 10^{-4}$, we have

$$\vec{V}_g \cong c_0 \frac{\vec{K}}{K}.$$

The discrepancy between the predictions of the generalized electromagnetic model and of the algorithm based on the relativistic kinematics is clearly expressed. The requirements indicated correspond to Fizeau experiment, if a moving rarefied gas is used in an experimental setup. According to the dynamical model of the electromagnetic field inertia, we can change the moving gas density so that bands in a Fizeau interferometer will begin to move. Such an experiment can be carried out any time.

2.3.3. The possibility to move with light velocity in vacuum for physical objects

At *w* = 1, the analysis of the dynamics of the transverse Doppler effect for the case of small relative velocities gives

$$\omega = \frac{\omega_0}{\left(1 - \frac{U_{fs}^2}{c^2}\right)^{1/2}}.$$

Let us multiply this expression be the quantity $\hbar/c^2$, where $\hbar$ is the Plank constant. Then we will obtain the dependence for masses which is used in the relativistic dynamics:

$$m = \frac{m_0}{\left(1 - \frac{U_{fs}}{c^2}\right)^{1/2}}.$$

It will be shown below that the generalized theory of electromagnetic phenomena gives another frequency formula when the velocities approach light speed in vacuum. Maintaining the relationship between frequency and mass valid, we will offer a new dependence of the mass on the velocity. For this purpose we maintain the above model of the radiation propagation from empty space in to the Earth's atmosphere, assuming that the velocity $\vec{U}_{fs}$ tends to the light velocity in vacuum. The problem can be easily solved entirely, but it is sufficient for our purposes to be restricted to a version when the value $w = 1$ is reached. Then $\vec{U} = 0$, $cK_z = n\omega_0$. Since U_{fs}/c is close to unity, the index of refraction corresponding to the actual situation is to be taken. Let $n = 1 + Q$, where $Q << 1$.

With allowance for the above remark, we obtain the following system of equations

$$c^2 K_x^2 = n^2\left(\omega^2 - \omega_0^2\right),\ \omega = \omega^0\left(1 - \frac{U_{fs}^2}{c^2}\right)^{1/2} + \frac{n}{c} U_{fs}\left(\omega^2 - \omega_0^2\right)^{1/2}.$$

The quadratic equation for the frequency

$$\omega^2 - 2\omega\omega_0\sigma\left(1 - \frac{U_{fs}^2}{c^2}\right)^{1/2} + \omega_0^2\sigma\left(1 + \frac{U_{fs}^2}{c^2}\Psi\right) = 0,$$

$$\Psi = 2Q + Q^2,$$

$$n = 1 + Q$$

now contains the factor

$$\sigma = \left[1 - U_{fs}^2 (1+\Psi)/c^2\right]^{-1}, \quad \Psi = 2Q + Q^2 .$$

The value of the field limited frequency is given by the law [3]

$$\omega = \omega_0 \sigma \left[\left(1 - \frac{U_{fs}^2}{c^2}\right)^{1/2} - \frac{U_{fs}^2}{c^2} \Psi^{1/2} (1+\Psi)^{1/2} \right].$$

It has no singularity for $U_{fs} \to c$. We obtain

$$\omega^* = \lim \omega \Big|_{U_{fs} \to c} = \omega_0 \left(1 + \frac{1}{\Psi}\right)^{1/2}.$$

Assuming that the mass is proportional to the frequency, we have the new formula

$$m = m_0 \frac{\left(1 - \frac{U^2}{c^2}\right)^{1/2} - \frac{U^2}{c^2} \Psi^{1/2} (1+\Psi)^{1/2}}{1 - \frac{U^2}{c^2}(1+\Psi)}.$$

The value of Ψ should be found from experiment.

2.3.4. The mechanical law of energy conservation for a photon.

When radiation propagates in a rarefied gas from a primary source, moving in vacuum with the velocity $\vec{U}_{fs}$, a dynamical change in its group velocity $\vec{V}_g$ and frequency ω occurs. At small relative velocities the frequency at the final stage of the dynamical process changes by the value

$$\omega - \omega_0 = 0{,}5\, \omega_0 \frac{U_{fs}^2}{c^2}.$$

Let us multiply this expression by the Plank constant $\hbar$ and use the Einstein formula for the photon inertia mass:

$$m_{in} = \hbar \frac{\omega_0}{c^2}.$$

This will yield the relation

$$\Delta U = E_{kin},$$

where the following designations are introduced:

a) the kinetic energy of the photon, which depends on the primary radiation source velocity

$$E_{kin} = 0{,}5\, \hbar \frac{\omega_0}{c^2} U_{fs}^2 ;$$

b) the potential energy of the photon, which depends on the frequency differences

$$\Delta U = \hbar(\omega - \omega_0).$$

The situation appears thus: the photon had the velocity $\vec{U}_{fs}$, additional to the light speed in vacuum c_0, and the frequency ω_0; in its interaction with the medium the velocity $\vec{U}_{fs}$ was "transformed" to the frequency ω.

Therefore the photon is similar to a physical body with its tangential $L_{\|}$ and transverse L_* lengths in the Newtonian space-time and it has an interior motion.

Let $L_* = a\lambda$, and $L_{\|} = b\lambda$, where *a* and *b* are constants.

Then the change of the frequency gives some changes in the L_* and $L_{||}$.

Conclusions

1. The generalization of Maxwell's electrodynamics, which takes into account all the forms of inertial motion, is possible, which, first, does not use the special relativity theory; second, it is based on the Newton space; third, gives superlight velocities and indicates the conditions under which they can be discovered; fourth, describes the known experimental facts, additionally assigning the dynamics of the external inertia parameters for the electromagnetic field.
2. The Bradly, Michelson, Fizo, and Doppler effects have a dynamic nature.
3. The special relativity theory correctly relates initial and final magnitudes of dynamical processes, fulfilling the function of a peculiar kind of a black box.
4. There is a dynamic mechanism of the transformation of the primary radiation source velocity into the electromagnetic field frequency because of its interaction with the medium, when the "mechanical" law of energy conservation is fulfilled.
5. The light speed in a moving rarefied gas can exceed the light speed in vacuum.
6. The velocity of an electromagnetic field in vacuum is not restricted to a limiting value, but for it to be measured it is necessary to take into account the interaction between the experimental devices and the field or those conditions, in which the field is spread.
7. The motion of particles for $m_0 \neq 0$ with the light speed in vacuum is possible.

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Part 3
Maxwell's electrodynamics without the velocity restriction in spinor form

RESUME. *It is shown, that Maxwell's electrodynamics without the velocity restriction has the spinor form for the matrix group V(4), which is* $(U(1)\times SU(2))\otimes(U(1)\times SU(2))$. *In this model, Newton's space-time is used with the Minkowski's space-time and with the Euclid's superlight space-time, which follow from the dynamic equations and the connections between the fields and inductions.*

Introduction

It is known that Maxwell's electrodynamics in vacuum has the spinor form [1-4]. The present work shows this form for the electrodynamics of the moving media. It shows that the electrodynamics model without SRT consists of the Newton's space-time as the base of the fibre bundle manifold, with $V(4)=(U(1)\times SU(2))\otimes(U(1)\times SU(2))$ as the fibre, and the Minkowski's space-time is used additionally to Euclid's super-light space-time.

3.1. Maxwell's electrodynamics in spinor form

Let us introduce

$$\Psi=\begin{pmatrix}E_x+iB_x\\ E_y+iB_y\\ E_z+iB_z\\ 0\end{pmatrix},\quad \varphi=\begin{pmatrix}H_x+iD_x\\ H_y+iD_y\\ H_z+iD_z\\ 0\end{pmatrix},\quad \Psi^*=\begin{pmatrix}E_x-iB_x\\ E_y-iB_y\\ E_z-iB_z\\ 0\end{pmatrix},\quad \varphi^*=\begin{pmatrix}H_x-iD_x\\ H_y-iD_y\\ H_z-iD_z\\ 0\end{pmatrix},$$

$$\partial_k=\left\{\partial_x,\partial_y,\partial_z,(-i)\frac{1}{c}\partial_t\right\},\quad \overset{*}{\partial_k}=\left\{\partial_x,\partial_y,\partial_z,i\frac{1}{c}\partial_t\right\},\quad U^k=\left\{\frac{U_x}{c},\frac{U_y}{c},\frac{U_z}{c},i\right\},$$

$$\overset{*}{U}{}^k=\left\{\frac{U_x}{c},\frac{U_y}{c},\frac{U_z}{c},-i\right\},\qquad g^{kn}=g_{kn}=diag(1,1,1,1),$$

$$r^{kn}=r_{kn}=diag(1,1,1,-1).$$

We will use

$$a^1=\begin{pmatrix}0&0&0&-1\\0&0&1&0\\0&-1&0&0\\1&0&0&0\end{pmatrix},a^2=\begin{pmatrix}0&0&-1&0\\0&0&0&-1\\1&0&0&0\\0&1&0&0\end{pmatrix},a^3=\begin{pmatrix}0&1&0&0\\-1&0&0&0\\0&0&0&-1\\0&0&1&0\end{pmatrix},$$

$$a^4=\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix},$$

$$b^1 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}, b^2 = \begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}, b^3 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix},$$

$$b^4 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix},$$

$$\Pi_1 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \qquad \Pi_2 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \qquad \Pi_3 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix},$$

$$\Pi_4 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

Then for F_{mn} and H_{mn} in the form

$$F_{mn} = \begin{pmatrix} 0 & B_z & -B_y & -iE_x \\ -B_z & 0 & B_x & -iE_y \\ B_y & -B_x & 0 & -iE_z \\ iE_x & iE_y & iE_z & 0 \end{pmatrix}, \quad H_{mn} = \begin{pmatrix} 0 & D_z & -D_y & iH_x \\ -D_z & 0 & D_x & iH_y \\ D_y & -D_x & 0 & iH_z \\ -iH_x & -iH_y & -iH_z & 0 \end{pmatrix}$$

we find

$$F_{mn} = \frac{i}{2}\left(a^k \Pi_k \Psi^* - b^k \Pi_k \Psi\right), \quad H_{mn} = \frac{-i}{2}\left(a^k \Pi_k \varphi - b^k \Pi_k \varphi^*\right).$$

The dynamic equations in spinor form can now be derived. The equations

$$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}, \quad \nabla \times \vec{B} = 0$$

are

$$\left\{\begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}\partial_x + \begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}\partial_y + \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{pmatrix}\partial_z + \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}\frac{(-i)}{c}\partial_t\right\} \times$$

$$\times\begin{pmatrix}E_x-iB_x\\E_y-iB_y\\E_z-iB_z\\0\end{pmatrix}+\left\{\begin{pmatrix}0&0&0&1\\0&0&1&0\\0&-1&0&0\\-1&0&0&0\end{pmatrix}\partial_x+\begin{pmatrix}0&0&-1&0\\0&0&0&1\\1&0&0&0\\0&-1&0&0\end{pmatrix}\partial_y+\begin{pmatrix}0&1&0&0\\-1&0&0&0\\0&0&0&1\\0&0&-1&0\end{pmatrix}\partial_z+\right.$$

$$\left.+\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix}\frac{i}{c}\partial_t\right\}\begin{pmatrix}E_x+iB_x\\E_y+iB_y\\E_z+iB_z\\0\end{pmatrix}=0.$$

We have analytically

$$a^k\,\partial_k\,\Psi^*+b^k\,\partial_k^*\,\Psi=0.$$

Let us introduce $\Phi=column\left(2\rho U_x,\,2\rho U_y,\,2\rho U_z,\,-2i\rho\right)4\pi$.

Then the equations

$$\nabla\times\vec{H}=\frac{1}{c}\frac{\partial\vec{D}}{\partial t}+4\pi\frac{\vec{j}}{c},\qquad\nabla\cdot\vec{D}=4\pi\rho$$

have the form

$$a^k\,\partial_k^*\,\varphi^*+b^k\,\partial_k\,\varphi=\Phi.$$

Let us write in spinor form the connections between the fields and inductions, proposed earlier:

$$\vec{B}+w\left[\vec{E}\times(\vec{u}/c)\right]=\mu\left(\vec{H}+\left[\vec{D}\times(\vec{u}/c)\right]\right),$$

$$\vec{D}+w\left[(\vec{u}/c)\times\vec{H}\right]=\varepsilon\left(\vec{E}+\left[(\vec{u}/c)\times\vec{B}\right]\right).$$

We deduce

$$i\mu\left\{\begin{pmatrix}0&0&0&1\\0&0&1&0\\0&-1&0&0\\-1&0&0&0\end{pmatrix}\frac{U_x}{c}+\begin{pmatrix}0&0&-1&0\\0&0&0&1\\1&0&0&0\\0&-1&0&0\end{pmatrix}\frac{U_y}{c}+\begin{pmatrix}0&1&0&0\\-1&0&0&0\\0&0&0&1\\0&0&-1&0\end{pmatrix}\frac{U_z}{c}+\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix}(-i)\right\}\times$$

$$\times\begin{pmatrix}H_x-iD_x\\H_y-iD_y\\H_z-iD_z\\0\end{pmatrix}-i\mu\left\{\begin{pmatrix}0&0&0&-1\\0&0&1&0\\0&-1&0&0\\1&0&0&0\end{pmatrix}\frac{U_x}{c}+\begin{pmatrix}0&0&-1&0\\0&0&0&-1\\1&0&0&0\\0&1&0&0\end{pmatrix}\frac{U_y}{c}+\begin{pmatrix}0&1&0&0\\-1&0&0&0\\0&0&0&-1\\0&0&1&0\end{pmatrix}\frac{U_z}{c}+\right.$$

$$\left.+\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix}(i)\right\}\begin{pmatrix}H_x+iD_x\\H_y+iD_y\\H_z+iD_z\\0\end{pmatrix}=w\left\{\begin{pmatrix}0&0&0&-w\\0&0&1&0\\0&-1&0&0\\w^{-1}&0&0&0\end{pmatrix}\frac{U_x}{c}+\begin{pmatrix}0&0&-1&0\\0&0&0&-w\\1&0&0&0\\0&w^{-1}&0&0\end{pmatrix}\frac{U_y}{c}+\right.$$

$$+\begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -w \\ 0 & 0 & w^{-1} & 0 \end{pmatrix}\frac{U_z}{c}+\frac{1}{w}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}(i)\Bigg\}\begin{pmatrix} E_x - iB_x \\ E_y - iB_y \\ E_z - iB_z \\ 0 \end{pmatrix}+w\Bigg\{\begin{pmatrix} 0 & 0 & 0 & w \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ w^{-1} & 0 & 0 & 0 \end{pmatrix}\frac{U_x}{c}+$$

$$+\begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & w \\ 1 & 0 & 0 & 0 \\ 0 & -w^{-1} & 0 & 0 \end{pmatrix}\frac{U_y}{c}+\begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & w \\ 0 & 0 & -w^{-1} & 0 \end{pmatrix}\frac{U_z}{c}+\frac{1}{w}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}(-i)\Bigg\}\begin{pmatrix} E_x + iB_x \\ E_y + iB_y \\ E_z + iB_z \\ 0 \end{pmatrix}=0.$$

If we introduce

$G_{kn} = diag(1, 1, 1, w^{-1})$, $R_{kn} = diag(1, 1, 1, -w^{-1})$, $\tilde{a}^k = Q^{-1} a^k Q$, $\Pi^k = g^{kn}\Pi_n$,

$Q^{-1} = diag(1, 1, 1, w)$, $\tilde{b}^k = Q^{-1} b^k Q$, $a_k = a^k$, $b_k = b^k$, $U_k = g_{kn} U^k$,
we can write

$$i\,\mu\left(b^k U_k^* \varphi^* - a^k U_k \varphi\right) = w G_{kn}\left(\tilde{a}^k U^n \Psi^* + \tilde{b}^k \overset{*}{U^n} \Psi\right)$$

and

$$i\,\varepsilon\left(b^k U_k \Psi^* - a^k U_k^* \Psi\right) = w R_{kn}\left(\tilde{a}^k U^{*n} \varphi^* + \tilde{b}^k U^n \varphi\right).$$

3.2. Analytic spinor form of the Maxwell's equations

If we use $w = 1$, we obtained the standard Maxwell's electrodynamics, deduced by Minkowski. Using our notations and

$$n^{ij} = diag(1, 1, 1, 0), \quad E_{ij} = diag(1, 1, 1, 1), \quad \overline{\Psi} = \Psi^*, \quad \overline{\varphi} = \varphi^*$$

we can write

$$g^{\alpha\beta} a_\alpha \partial_\beta \left(E_{ij} n^{ij} \overline{\Psi}\right) + r^{\alpha\beta} b_\alpha \partial_\beta \left(E_{ij} n^{ij} \Psi\right) = 0,$$

$$r^{\alpha\beta} a_\alpha \partial_\beta \left(E_{ij} n^{ij} \overline{\varphi}\right) + g^{\alpha\beta} b_\alpha \partial_\beta \left(E_{ij} n^{ij} \varphi\right) = \Phi,$$

$$i\mu\left(r_{\alpha\beta} b^\alpha U^\beta E_{ij} n^{ij} \overline{\varphi} - g_{\alpha\beta} a^\alpha U^\beta E_{ij} n^{ij} \varphi\right) = g_{\alpha\beta} a^\alpha U^\beta E_{ij} n^{ij} \overline{\Psi} + r_{\alpha\beta} b^\alpha U^\beta E_{ij} n^{ij} \Psi,$$

$$i\varepsilon\left(g_{\alpha\beta} b^{\alpha} U^{\beta} E_{ij} n^{ij} \overline{\Psi} - r_{\alpha\beta} a^{\alpha} U^{\beta} E_{ij} n^{ij} \Psi\right) == r_{\alpha\beta} a^{\alpha} U^{\beta} E_{ij} n^{ij} \overline{\varphi} + g_{\alpha\beta} b^{\alpha} U^{\beta} E_{ij} n^{ij} \varphi,$$

$$F_{\alpha\beta} = \frac{i}{2} g_{\alpha\gamma} g_{\beta\delta} \left(a^{\gamma} \Pi^{\delta} \overline{\Psi} - b^{\gamma} \Pi^{\delta} \Psi\right),$$

$$H_{\alpha\beta} = \frac{-i}{2} g_{\alpha\gamma} g_{\beta\delta} \left(a^{\gamma} \Pi^{\delta} \varphi - b^{\gamma} \Pi^{\delta} \overline{\varphi}\right).$$

Here we have not only the Newton's space with n^{ij}. The equations have the Minkowski's space with g^{ij} and the Euclid's space with r^{ij}.

3.3. Fundamental group for the physics

We will use the Pauli group $V(2) = U(1) \times SU(2)$, where

$$\sigma^0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

Let us introduce $V(4) = V(2) \otimes V(2)$ in form [5]:

$$\sigma_0^0 = \overbrace{\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}}^{E} \quad \sigma_0^1 = \overbrace{\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}}^{e^3} \quad \sigma_0^2 = \overbrace{\begin{pmatrix} 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \\ 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \end{pmatrix}}^{b^3} \quad \sigma_0^3 = \overbrace{\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}}^{c^1}$$

$$\sigma_1^0 = \overbrace{\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}}^{e^2} \quad \sigma_1^1 = \overbrace{\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}}^{e^1} \quad \sigma_1^2 = \overbrace{\begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \end{pmatrix}}^{a^1} \quad \sigma_1^3 = \overbrace{\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}}^{f^2}$$

$$\sigma_2^0 = \overbrace{\begin{pmatrix} 0 & 0 & -i & 0 \\ 0 & 0 & 0 & -i \\ i & 0 & 0 & 0 \\ 0 & i & 0 & 0 \end{pmatrix}}^{a^2} \quad \sigma_2^1 = \overbrace{\begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & -i & 0 \\ 0 & i & 0 & 0 \\ i & 0 & 0 & 0 \end{pmatrix}}^{b^1} \quad \sigma_2^2 = \overbrace{\begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}}^{f^1} \quad \sigma_2^3 = \overbrace{\begin{pmatrix} 0 & 0 & -i & 0 \\ 0 & 0 & 0 & i \\ i & 0 & 0 & 0 \\ 0 & -i & 0 & 0 \end{pmatrix}}^{b^2}$$

$$\sigma_3^0 = \overbrace{\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}}^{c^3} \quad \sigma_3^1 = \overbrace{\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \end{pmatrix}}^{f^3} \quad \sigma_3^2 = \overbrace{\begin{pmatrix} 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \\ 0 & 0 & 0 & i \\ 0 & 0 & -i & 0 \end{pmatrix}}^{a^3} \quad \sigma_3^3 = \overbrace{\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}}^{c^2}.$$

The elements (a^1, a^2, a^3, E), (b^1, b^2, b^3, E) form the sub algebras with the conditions $\xi^i\xi^j - \xi^j\xi^i = c_k^{ij}\xi^k = [\xi^i, \xi^j]$. The elements (c^1, c^2, c^3, E), (e^1, e^2, e^3, E), (f^1, f^2, f^3, E) are the sub algebras with the conditions $\xi^i\xi^j + \xi^j\xi^i = c_k^{ij}\xi^k = \{\xi^i, \xi^j\}$. Let us introduce $\sigma = -1$ for the elements E, a^i, b^i and $\sigma = 1$ for the elements c^i, e^i, f^i. For any elements from $V(4)$ we have the conditions

$$\xi^i\xi^j + \sigma_{(i)}\sigma_{(j)}\sigma_{(k)}\xi^j\xi^i = c_k^{ij}\xi^k,$$

$$\xi^i\xi^j\xi^k + \sigma_{(i)}\sigma_{(j)}\sigma_{(k)}\sigma_{(l)}\xi^k\xi^j\xi^i = c_l^{ijk}\xi^l,$$

$$\xi^i\xi^j\xi^k\xi^l + \sigma_{(i)}\sigma_{(j)}\sigma_{(k)}\sigma_{(l)}\sigma_{(m)}\xi^l\xi^k\xi^j\xi^i = c_m^{ijkl}\xi^m, \ldots .$$

which determinate a new algebra. The designation (i) means the absence of the summation on the coinciding indexes. We understand now, that Maxwell's electrodynamics without *SRT* has the Newton's space-time as the base of the fibre bundle manifold and $V(4)$ as the fibre [6]. We can give a simple "picture" of the group $V(4)$. Really, the elements of the subgroups have its own places, according figure1:

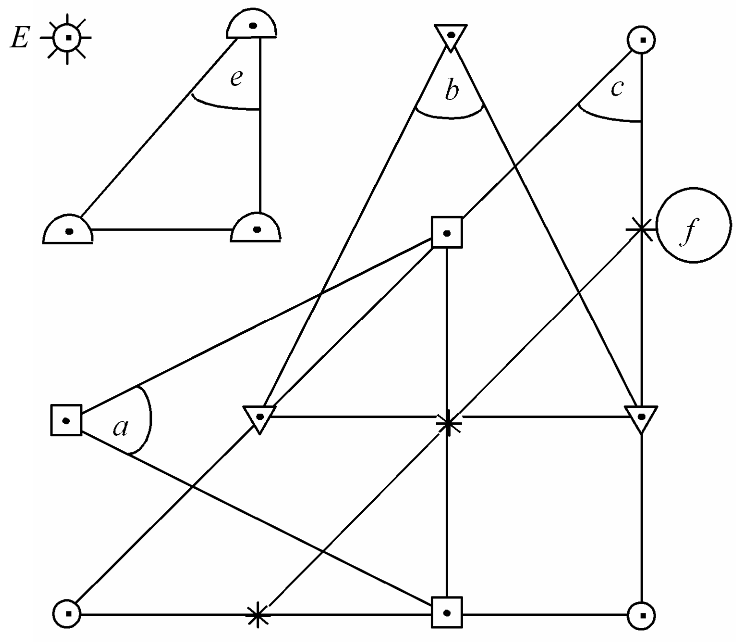

Fig.1. The picture of the group V (4).

Conclusions

We have shown, that Maxwell's electrodynamics without the velocity restriction has a simple spinor form for the group $V(4)$. We see, that in this form, the Minkowski's space-time and the Euclid's space-time are used additionally to the Newton's space-time.

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Part 4
Physical mechanism of the dynamical transformation of the field inertia for Maxwell's electrodynamics without the velocity restriction

RESUME. *This work suggests a physical mechanism of the dynamical transformation of the field inertia for Maxwell's electrodynamics without the velocity restriction. It is found, that two scalar cohomological groups govern the field inertia.*

Introduction

Part 1 and Part 2 of this series suggests a new model of the field inertia, comprising two different parts: proper inertia, which depends on the index of the refraction $n(\rho)$, and external inertia, which depends on the new magnitude $w(\rho)$, named the «phase inertia». In this model, the velocity and the frequency of the field dynamically depend on $n(\rho)$ and $w(\rho)$. Earlier, only simple solutions for the proposed model were derived. Now some new exact solutions, based on a generalized Green's function, will be derived. We will study the physical mechanism of the change of the proper and external field inertia.

4.1. Generalized Green's function

Following from the new model, we have the Green's function for the vector equation

$$\left\{\left(\Delta-\frac{w}{c^2}\frac{\partial^2}{\partial t^2}\right)-\frac{\varepsilon\mu-w}{c^2}\frac{1}{\left(1-w\beta^2\right)}\left(\frac{\partial}{\partial t}+\vec{u}\cdot\nabla\right)^2\right\}\vec{A}=0,$$

in the form [1]

$$G_0(\vec{r},t)=16\pi^4\mu(r^2+\xi^2)^{-1/2}\delta\left(t-\frac{1}{c}\frac{\varepsilon\mu-\beta^2w^2}{(1-w\beta^2)\sqrt{\varepsilon\mu}}\left(r^2+\xi^2\right)^{1/2}\right).$$

Here $\vec{u}=\left(1-w\right)\vec{u}_{fs}+w\vec{u}_m$, $\vec{u}_m$ is the matter velocity, $\vec{u}_{fs}$ is the first source velocity,

$$\vec{\beta}=\vec{u}/c,\ \xi=z-\frac{\varepsilon\mu-w}{\varepsilon\mu-\beta^2w^2}u\,t,\quad r^2=\rho^2\frac{\varepsilon\mu(1-w\beta^2)}{\varepsilon\mu-\beta^2w^2}.$$

In a cylindrical coordinate system, the position vector has length

$$R=\left(\rho^2+z^2\right)^{\frac{1}{2}}.$$

We will analyze some exact solutions transforming the Green's function. In accordance with standard method [2], we have the relation for the δ-function

$$\delta(f(t))=\sum_s\frac{\delta(t-t_s)}{\left|f'(t_s)\right|},$$

where

$$f(t) = t - \frac{1}{c}\frac{\varepsilon\mu - \beta^2 w^2}{(1 - w\beta^2)\sqrt{\varepsilon\mu}}\left\{\rho^2 \frac{\mu(1 - \mu\beta^2)}{\varepsilon\mu - \beta^2 w^2} + \left(z - \frac{\varepsilon\mu - w}{\varepsilon\mu - \beta^2 w^2} ut\right)^2\right\}^{1/2}, \qquad f'(t) = \frac{df}{dt}.$$

We will introduce

$$a = \frac{(\varepsilon\,\mu - \beta^2 w^2)c^{-1}}{(1 - w\,\beta^2)\sqrt{\varepsilon\,\mu}}, \qquad b = \frac{\varepsilon\,\mu - w}{\varepsilon\,\mu - \beta^2 w^2}.$$

The roots t_s are

$$t_{1,2} = \frac{1}{c}\frac{-(\varepsilon\mu - w)\beta\, z \pm \sqrt{\varepsilon\mu}(1 - w\beta^2)[z^2 + \rho^2(1 - \varepsilon\mu\beta^2)/(1 - w\beta^2)]^{1/2}}{1 - \varepsilon\mu\beta^2}.$$

The derivates $|f'(t_s)|$ are equal. So we have

$$|f'(t_1)| = |f'(t_2)| = a\left(z^2 + b(1 - a^2 z^2)\right)^{1/2}.$$

With the function

$$\operatorname{sgn} a = \frac{a}{|a|} = \begin{cases} 1, & a > 0, \\ -1, & a < 0 \end{cases},$$

we have

$$G_0(\vec{r}, t) = 16\pi^4 \mu \frac{0.5(1 + \operatorname{sgn} t_1)\delta(t - t_1) + 0.5(1 + \operatorname{sgn} t_2)(\delta(t - t_2))}{[z^2 + [(1 - \varepsilon\,\mu\,\beta^2)/(1 - w\,\beta^2)]\rho^2]^{1/2}}.$$

We can analyze now some particular situations for different phase velocities $\vec{v}_f = \frac{\omega}{k}\vec{s}$, where ω is the frequency, $\vec{k}$ is the wave vector.

1. For $v_f < c/\sqrt{\varepsilon\mu}$, we have

$$G_0(\vec{r}, t) = 16\pi^4 \mu\, \delta(t - t_1)\left(z^2 + \frac{1 - \varepsilon\,\mu\,\beta^2}{1 - \beta^2 w}\rho^2\right)^{-1/2}.$$

2. For $v_f = c/\sqrt{\varepsilon\mu}$, we have

$$t_1 = \frac{\sqrt{\varepsilon\mu}}{2c}\left[(1 + \frac{w}{\varepsilon\mu})z + \frac{\rho^2}{2}\right], \qquad t^2 = \infty$$

and

$$G_0(\vec{r}, t_1) = \frac{16\pi^4 \mu}{z}\delta(t - t_1).$$

3. For $v_f > c/\sqrt{\varepsilon\mu}$, we have

$$G_0(\vec{r}, t) = 16\pi^4 \mu\left[\delta(t - t_1) + \delta(t - t_2)\right]\left(z^2 + \frac{1 - \varepsilon\mu\beta^2}{1 - \beta^2 w}\rho^2\right)^{-1/2}.$$

Two roots t_1 and t_2 are positive. The surface of the wave front is a cone with the angle

$$tg\,\Theta_g = \left(\frac{1 - w\beta^2}{\varepsilon\,\mu\beta^2 - 1}\right)^{1/2},$$

which has nonlinear dependence on w.

4.2. Geometrical optics approximation

Consider the situation in which the phase inertia w changes slowly. For small velocities, when $\beta^2 << 1$, we have

$\vec{B} = \mu \vec{H} + [\vec{G} \times \vec{E}], \quad \vec{D} = \varepsilon \vec{E} - [\vec{G} \times \vec{H}]$,

where $\vec{G} = -(\mu\varepsilon - w)\vec{\beta}$.

The light ray is describing by the dispersion equation [3]

$(\vec{k} - \vec{G})^2 = n^2$,

where $\vec{k} = \nabla \psi$. The Hamiltonian is

$H = 0.5\left[(\vec{K} - \vec{G})^2 - n^2\right]$.

From the Hamilton-Jacoby equations we see, that the vector $d\vec{r}/ds$ depends on $\vec{k}$ and $\vec{G}$. $\vec{G}$ is nonlinear function of $w(\rho)$.

4.3. Cohomological mechanism of the field inertia

We will determine the proper field inertia for the case when $\vec{u}_m = \vec{u}_{fs} = 0$. Then the group velocity is

$$\vec{v}_g = \frac{c}{n}\frac{\vec{k}}{k}$$

and the rate of the refraction n governs the field inertia. We will determine the external field inertia for the case when $\vec{u}_m \neq 0$, $\vec{u}_{fs} \neq 0$. Then the group velocity has the form

$$\vec{v}_g = \frac{c}{n}\frac{\vec{k}}{k} + \left(1 - \frac{w}{n^2}\right)\left[(1-w)\vec{u}_{fs} + w\vec{u}_m\right].$$

We will discuss the physical and mathematical meanings of the magnitudes n and w, using the general form of the connection between fields and inductions. Maxwell's generalized equations are $\partial_{[k} F_{mn]} = 0$, $\partial_k H^{ik} = S^i$, $H^{ik} = \Omega^{im}\Omega^{kn}F_{mn}$ with

$$\Omega^{ij} = \frac{1}{\sqrt{\mu}}\left[\Theta^{ij} + \left(\frac{\varepsilon\mu}{w} - 1\right)u^i u^j\right],$$

$$\Theta^{ij} = diag\,(1, 1, 1, w), \quad u^i = dx^i / d\Theta.$$

If the velocity is $\vec{u}_m = \vec{u}_{fs} = 0$, then $u^i = diag\,(0, 0, 0, \sqrt{w})$ and

$$\overset{(0)}{\Omega^{00}}\Big|_{\vec{u}=0} = \frac{1}{\sqrt{\mu}}\left[w + \left(\frac{\varepsilon\mu}{w} - 1\right)w\right] = \varepsilon\sqrt{\mu},$$

$$\overset{(0)}{\Omega}{}^{11} = \overset{(0)}{\Omega}{}^{22} = \overset{(0)}{\Omega}{}^{33} = \frac{1}{\sqrt{\mu}}.$$

We have $\overset{(0)}{\Omega}{}^{ij} = diag(1, 1, 1, \varepsilon\mu)\frac{1}{\sqrt{\mu}}$. We introduce now two magnitudes:

a) $\sigma = \det \overset{(0)}{\Omega}{}^{ij} \Big/ \det \overset{(0)}{\Omega}{}^{ij}_{*}$;

b) $w = \det \Theta^{ij} / \det \Theta^{ij}_{*}$,

where $\overset{(0)}{\Omega}{}^{ij}_{*} = diag(1, 1, 1, 1) = \Theta^{ij}_{*}$.

So we have in Maxwell's electrodynamics two scalar functions, which form two scalar co-homological groups $H^0(G, A)$ [4]. Really, for any group $g \in G$ and any scalar elements $a \in \sigma$, w we receive the condition $g\,a = a$ and $a \in H^0(G, A)$. This means that the dynamic of the electromagnetic field inertia are governed by two co-homological groups $H^0(G, A)$: $a \in \sigma$, $a \in w$.

Conclusions

Now we understand that the dynamic change of the electromagnetic field inertia is based on two 0-cohomological groups: one of them governs the proper field's inertia and the other governs the external field's inertia.

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Part 5
ACTIVE SYMMETRIES AND ISOMETRIES OF THE MAXWELL'S ELECTRODYNAMICS

RESUME. *It is shown, that Maxwell's electrodynamics without the velocity restriction has for the rest and moving media an isometry group which is the Lorentz group with generators and parameters depending on its 0-cohomologies, governing the dynamics of the frequency and the velocity of an electromagnetic field in Newtonian space-time.*

Introduction

In the first and second parts of this series the generalized connections between fields and inductions in Maxwell's electrodynamics for moving media are proposed. They have allowed agreeing experimental dates with the solutions of the equations within the framework of Newtonian space-time, without use of the special relativity theory. In this article the local isometric symmetries for offered combined equations are studied. It is shown, that they depend on 0-cohomologies in such a manner that the dynamical changes of the frequency and the field velocity are in concordance with the Galilee and the Lorentz group, which are physically supplement. They operate in tangential space with the local metric, associated with the connections between fields and inductions both in the rest and the moving media. Newton's space at such approach is the base of the fibre bundle manifolds, which remains invariable, if the isometry group is acting in the fibre.

5.1. Active symmetries of Maxwell equations

It is known, that the Maxwell's tensor equations - $\partial_{[k} F_{mn]} = 0$, $\partial_k H^{ik} = S^i$ are invariant for the 20-parameter Lie algebra of the group $IGL(4, R)$, containing the Poincare sub algebra $AP(1,3)$ and the Galilei sub algebra $AG(1,3)$ [1]. The simple proof of this fact we can find in the Post's book [2]. It is obvious enough, because the Maxwell's equations represent linear expressions for tensors, which are derivate from tensors of the second rank F_{mn}, H^{ik} and the covector derivatives. Schouten has shown [3], that viewed system has general covariant if covariant, derivatives take the place of partial derivatives. Moreover it is only one, if there are no other fields. The tensor connection for fields and inductions in form $H^{ik} = \chi^{ikmn} F_{mn}$ has the general covariance symmetry by virtue of the tensor definition. It is clear, that an invariance of the equations is not enough for the physical analysis, if the group is so large. The situation varies, when the requirement, that the symmetry is an isometry, is added. Then the coordinates and time transformations are considering, for which the symmetric tensor g_{ij}, viewed as the metric of tangential space with the interval $ds^2 = g_{ij} dx^i dx^j$ is invariable. In this case the interval has the same form in different coordinate frames. From the topological point of view the isometry requirement means, that the 0-cohomologies of the metric are identical. We have the expressions $a = \det(\xi)$, $b = sp(\xi)$, where (ξ) is the metric tensor of the tangential space, given in the matrix form. As we are interested in solution for concrete conditions it is naturally to select these solutions from the analysis of the connections between fields and inductions for the electromagnetic field. We will study isometries for the rest and the moving media.

5.2. Isometries in the rest media

Let $\vec{D} = \varepsilon\vec{E}$, $\vec{B} = \mu\vec{H}$. The connection $H^{ik} = \overset{(0)}{\Omega}{}^{im} \overset{(0)}{\Omega}{}^{kn} F_{mn}$ gives the tensor $\overset{(0)}{\Omega}{}^{im} = \frac{1}{\sqrt{\mu}} diag\left(1, 1, 1, \varepsilon\mu\right)$.

We will find the isometry group for the condition

$\overset{(0)}{\Omega}{}_{i'j'}\, dx^{i'} dx^{j'} = \overset{(0)}{\Omega}{}_{ij}\, dx^i dx^j$,

when Ω_{ij} is identical in hatched and not hatched coordinate frames. Let us note, that ε, μ depend on Newton's space coordinates and Ω_{ij} too. But for tangential space this dependence is parametrical, because we study the connection between $dx^{\mu}{}'$ and dx^{μ}. It is easy to see, that coordinate transformations

$$dx' = \frac{dx - vdt}{\left(1 - \varepsilon\mu\frac{v^2}{c^2}\right)^{1/2}}, \quad dy' = dy, \quad dz' = dz, \quad dt' = \frac{dt - \varepsilon\mu\frac{v}{c^2}dx}{\left(1 - \varepsilon\mu\frac{v^2}{c^2}\right)^{1/2}}$$

are isometries (at $v = const$) because

$$dx'^2 + dy'^2 + dz'^2 - \frac{1}{\varepsilon\mu}c^2 dt'^2 = dx^2 + dy^2 + dz^2 - \frac{1}{\varepsilon\mu}c^2 dt^2.$$

We note, that at $\varepsilon\mu = 1$ we receive the Lorentz transformations. For any other values ($\varepsilon\mu \neq 1$) there are other values of "maximum velocities", equal $c^* = c/n$. By direct substitution, following [4], easily to prove that Maxwell's equations maintain its form at the transformations with $\varepsilon\mu = const$. Then the connections $\vec{D} = \varepsilon\,\vec{E}$, $\vec{B} = \mu\,\vec{H}$ are transformed to $\vec{D}' = \varepsilon\,\vec{E}'$, $\vec{B}' = \mu\,\vec{H}'$ in accordance with the rules:

$$E'_x = E_x,\ E'_y = \gamma\left(E_y - \frac{v}{c}B_z\right),\ E'_z = \gamma\left(E_z + \frac{v}{c}B_y\right),$$

$$B'_x = B_x,\ B'_y = \gamma\left(B_y + \frac{v}{c}\varepsilon\mu\,E_z\right),\ B'_z = \gamma\left(B_z - \frac{v}{c}\varepsilon\mu\,E_y\right),$$

$$D'_x = D_x,\ D'_y = \gamma\left(D_y + \frac{v}{c}\varepsilon\mu\,H_z\right),\ D'_z = \gamma\left(D_z - \frac{v}{c}\varepsilon\mu\,H_y\right),$$

$$H'_x = H_x,\ H'_y = \gamma\left(H_y + \frac{v}{c}D_z\right),\ H'_z = \gamma\left(H_z - \frac{v}{c}D_y\right),$$

$$\gamma = \left(1 - \varepsilon\mu\frac{v^2}{c^2}\right)^{1/2},$$

which confirm our conclusion. In this approach the isometry group operates in tangential space with the metric $\overset{(0)}{\Omega}{}_{ij}$, associated with the connections between fields and inductions. We have therefore the space of events *SE*. It is locally Riemannian space. The Newton's space, in which one we are describing the electromagnetic phenomena, is self-dependent and forms the space of state *SS*. So, we will adopt the point of view, that Maxwell's electrodynamics is based on two spaces, which are independent. As it is enough to have such model for the explanation of the experimental facts, we understand, that the pair of spaces (*SE* and *SS*) indicated there is enough too. However it is only physical conclusion, but its mathematical essence remains vague. Also the equations are invariant

$$\vec{D} + \varepsilon\mu\left[\vec{\beta} \times \vec{H}\right] = \varepsilon\left(\vec{E} + \left[\vec{\beta} \times \vec{B}\right]\right),$$

$$\vec{B} + \varepsilon\mu\left[\vec{E} \times \vec{\beta}\right] = \mu\left(\vec{H} + \left[\vec{D} \times \vec{\beta}\right]\right),$$

$$\vec{\beta} = \vec{u}/c\,.$$

There are no inconsistencies with the previous deduction, because, as it is easy to see, the "convective" terms of the connections are canceling each other.

5.3. Isometries in the moving media

At the beginning of this series it is shown, that the connections $\vec{D} = \varepsilon\,\vec{E}$, $\vec{B} = \mu\,\vec{H}$ can be extended for the case when the medium has the velocity $\vec{u}_m$ and a primary source has the velocity $\vec{u}_{fs}$. For this purpose we use the velocity $\vec{u} = (1-w)\vec{u}_{fs} + w\vec{u}_m$ and also the tensor $g^{ij} = diag\,(1, 1, 1, w)$. We deduced the tensor

$$\Omega^{ij} = \frac{1}{\sqrt{\mu}}\left[g^{ij} + \left(\frac{\varepsilon\mu}{w} - 1\right)u^i u^j\right],$$

connecting the fields and inductions in the moving media.

Let us show, that the similar expressions follow from the assumption, that the tensor g_{ij}, depending from w, is isomeric in the tangential space of events *SE*. We will receive for the metric g_{ij} the isometry (similarly to the isometries in the rest media) in the form:

$$dx' = \frac{dx - vdt}{\left(1 - w\frac{v^2}{c^2}\right)^{1/2}}, \quad dy' = dy\,, \quad dz' = dz\,, \quad dt' = \frac{dt - w\frac{v}{c^2}dx}{\left(1 - w\frac{v^2}{c^2}\right)^{1/2}}.$$

Now it is easy to find the fields and inductions transformations concerning this isometry. The connections, which are invariant for the isometry transformation, receive the next form:

$$\vec{D} + w\left[\vec{\beta} \times \vec{H}\right] = \varepsilon\left(\vec{E} + \left[\vec{\beta} \times \vec{B}\right]\right),$$
$$\vec{B} + w\left[\vec{E} \times \vec{\beta}\right] = \mu\left(\vec{H} + \left[\vec{D} \times \vec{\beta}\right]\right).$$

Earlier we deduced that these connections correspond to model of the dynamical description of the relativistic effects in Maxwell's electrodynamics without special relativity theory. In this approach the isometry can be used not only for the kinematic explanation of the class of the equivalent solution, as it is accepted in special relativity theory, but for a deduction of the equations, connecting fields and inductions. Such approach has been offered initially by Minkowski [5]. Let us express $\vec{B}$ and $\vec{D}$ through $\vec{E}$ and $\vec{H}$. Then

$$\vec{B} = \frac{1}{1-\varepsilon\mu\beta^2}\left\{\mu\left(1-w\beta^2\right)\vec{H} + (\varepsilon\mu - w)\left[\left(\vec{E}\times\vec{\beta}\right) - \mu\vec{\beta}\left(\vec{\beta}\cdot\vec{H}\right)\right]\right\},$$

$$\vec{D} = \frac{1}{1-\varepsilon\mu\beta^2}\left\{\varepsilon\left(1-w\beta^2\right)\vec{E} + (\varepsilon\mu - w)\left[\left(\vec{\beta}\times\vec{H}\right) - \varepsilon\vec{\beta}\left(\vec{\beta}\cdot\vec{H}\right)\right]\right\}.$$

From the obtained relations follows, that at $w = \varepsilon\mu$ we receive the isometries for the rest media and the equations $\vec{D} = \varepsilon\,\vec{E}$ and $\vec{B} = \mu\,\vec{H}$. The isometries, depending on w, if to distract from the method by which one they are obtained, represent the symmetry class, which is parametrically dependent from w. But, as it was indicated earlier, the quantities

$a = \det\left(g^{ij}\right), \quad b = sp\left(g^{ij}\right),$

give the 0-cohomologies, introduced by Hochschild [6]. They do not vary at the isometry transformations because of the invariance of the metric tensor. It is known, that the cohomologies characterize the topological properties of the phenomena. In this case they are expressed by $w(x)$. At $w = 0$ we have Galilee's symmetry. At $w = 1$ we receive the canonical Lorentz's group. In general case we use expression $w = 1 - \exp\left[-P_0\,\rho/\rho_0\right]$, where P_0 - relaxation constant, ρ, ρ_0 - densities of the media. Therefore the change of the quantity ρ gives the change of the local isometry corresponding w. In the dynamic process for the electromagnetic fields the frequency change and the velocity change are governing by the Galilee's and Lorentz's groups. The isometry bears no relation to the structure of Newtonian space, in which one the phenomena are considered. Therefore it is possible to construct the fibre bundle manifold with the pair of the spaces *SE* and *SS* [7]. Our situation is standard for such approach. It is clear, that we can use the Newtonian's space as the base of this manifold and the isometry group as its fibre. So we receive the models widely used in the differential geometry.

5.4. 0-cohomologicaly dependent active Lorentz transformations

Let us show the algorithm, following to which one, we can receive generalized transformations, depending on w, using the canonical Lorentz transformations. In the first place we will use the Lorentz transformations in the infinitesimal form

$g = I + A_s\omega^s,$

where A_s are the generators of the symmetry, ω^s are the parameters of the symmetry.

Let us associate with the Lorentz symmetry the 0-cohomology group, consisted of the scalar functions $w(x)$. Analogical functions can be introduced into the physical theory by different ways. The magnitudes $w(x)$ in the interval (0÷1] form the multiplicative Abelian group. It is obviously, that the representation of this group is $Q^{-1} = diag\left(1, 1, 1, w\right)$. Now we will modificate the canonical Lorentz transformations. Let

$\widetilde{A}_s = Q^{-1}A_sQ, \quad \widetilde{u}^s/c = (1-w)u^s_{fs}/c + w\,u^s_m/c.$

Here we have, apparently, the class of the algebraically equivalent generators of the symmetry and the class homotopically equivalent parameters of the symmetry. So we receive 0-cohomologicaly dependent transformations

$g = I + \widetilde{A}_s\widetilde{\omega}^s.$

Using this method, we deduce the generalized generators of the Lorentz group in the form

$$\tilde{A}_4 = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & w \end{vmatrix} \begin{vmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{vmatrix} \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & w^{-1} \end{vmatrix} = \begin{vmatrix} 0 & 0 & 0 & -w^{-1} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ w & 0 & 0 & 0 \end{vmatrix} \ldots$$

They give the isometry group for the moving media, which was described above. It depends on the 0-cohomologies and consequently it is necessary to consider it as the 0-cohomologicaly dependent Lorentz transformations. Using the change of w we attempt to take into account the topological aspects the electromagnetic field. For this reason we can interpreted the dynamic change of the frequency and the velocity of the field as the change of its topological aspects.

Conclusions

The 0-cohomologicaly dependent transformations of the coordinates, which are expressed by the isometries in the rest and moving media, permit to describe the dynamic states of the electromagnetic field interacting with medium. The symmetry of the vacuum equations is given by canonical Lorentz transformations and it is the isometry group. Using 0-cohomologies and proposed algorithm of the symmetry modification, it is possible to find all class of the isometries and the equations for the moving media.

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Part 6
The dynamical influence of the classical experimental equipment at the electromagnetic field

RESUME. *The dynamical influence of the classical experimental equipment at the electromagnetic field is described by the class of the Lorentz's transformations dependent on 0-cohomology group* $w(x)$. *They act in the space of events SE, which is additional to the Newton's space.*

Introduction

In part 1 and part 2 of this series we have shown, that Maxwell's electrodynamics with the generalized connections between the fields and inductions gives the dynamical description of the relativistic experiments in Newton's space-time without using the special relativity theory. This possibility is based mathematically on the dynamics of the 0-coho-mology group with the elements $w(x)$, which govern the behavior of the field's parameter. This possibility is based physically on proposed new property of the electromagnetic field: at the dynamical change of its external inertia. Part V of this series suggests an isomerty group for the moving media in the form of the Lorentz's transformations dependent on the 0-cohomology group, which acts in the space of the events SE. This dependence allows giving the dynamical explanation of the relativistic experiments, using the space-time transformations, concerning the structure of the space of the events *SE*. So we receive the generalized kinematic method, which we can use additionally to standard physical description. Really, it is known, that the symmetry of the equations, describing the physical events, gives the class of the model solutions. It is useful, for this reason, to consider the class of parametrical symmetries, if we want to compare the different experimental results.

We can use this method in the real practice. We will suppose at first, that the experimental equipment is the real medium (with the special concrete construction) and, secondly, that the measuring is the interaction of the experimental equipment with the electromagnetic field. We will describe the experimental equipment by means of the rate of the refraction $n_d(x)$ and the velocity $\vec{u}_d$, which is analogous to the medium velocity $\vec{u}_m$. We will use the phase inertia $w_d(x)$, which describes the dynamical change of the external field's inertia. This method allows us to give two ways for the description of the interaction of the experimental equipment with the electromagnetic field. First way includes the solution of the Maxwell's electrodynamics with the generalized connections between the fields and inductions with the parameters $n_d(x)$, $u_d(x)$, $w_d(x)$. Second way includes the generalized space-time transformations with the parameters w_d, $\vec{u}_d$. They act in the space of the events SE, which is associated with the connections between the fields and inductions.

6.1. Lorentz's transformations dependent on the 0-cohomology group

We will modificate the infinitesimal Lorentz's transformations

$g = I + A_s \omega^s$,

using the 0-cohomology group G with the elements $w(x)$, which is related with the

rate of the refraction $n(x)$ by the equation [1]:
$w = 1 - \exp[-P_0(n-1)]$.
We will use the magnitude
$Q^{-1} = diag(1, 1, 1, w)$
as the representation of the 0-cohomology group G. We will transform the generators A_s, using the rule
$\widetilde{A}_s = Q^{-1} A_s Q$.
For example, we will receive

$$\widetilde{A}_4 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & w \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & w^{-1} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & -w^{-1} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ w & 0 & 0 & 0 \end{pmatrix}.$$

Let the parameters of the Lorentz's group ω^s are invariable. We will derive the generalized space-time transformations, according the equations
$dx^{\mu}{}' = \left(I + \widetilde{A}_S\, \omega^S\right)^{\mu}_{\nu} dx^{\nu}$.
Let us introduce $\omega^4 = \Theta$. We have deduced, that
$dx^1{}' = dx^1 - w^{-1}\Theta\, dx^0$, $dx^0{}' = dx^0 + w\Theta dx^1$, $dx^2{}' = dx^2$, $dx^3{}' = dx^3$.
These transformations for $\Theta >> \varepsilon$ have the form
$dx^1{}' = dx^1 \cos\Theta - icdtw^{-1} \sin\Theta$, $dx^2{}' = dx^2$,
$dt' = dt\cos\Theta - \frac{i}{c} dxw\sin\Theta$, $dx^3{}' = dx^3$,
where
$x^1 = x$, $x^2 = y$, $x^3 = z$, $x^0 = ict$.
Let us introduce the coordinate system with the velocity *v*, which corresponds to the condition $dx^1{}' = 0$:
$tg\Theta = -i\, w\frac{v}{c}$.
We receive
$\cos\Theta = \left(1 - w^2 \frac{v^2}{c^2}\right)^{-1/2}$, $\sin\Theta = \pm i\frac{v}{c} w\left(1 - w^2 \frac{v^2}{c^2}\right)^{-1/2}$
and the space-time transformations

$$dx' = \frac{dx - vdt}{\left(1 - w^2 \frac{v^2}{c^2}\right)^{1/2}},\ dy' = dy,\ dz' = dz,\ dt' = \frac{dt - w^2 \frac{v}{c^2} dx}{\left(1 - w^2 \frac{v^2}{c^2}\right)^{1/2}}.$$

For *w*=0 we have the Galilee's group, for *w*=1 we have the canonical Lorentz's group. In general case these transformations describe the action of the Lorentz's group, dependent on 0-cohomology group $w(x)$, in the tangential space. When the magnitude $w(x)$ changes, then the space-time transformations give the dynamical change of the magnitudes $\{dx^k{}'\}$, if we consider $dx^k = const$. This approach is different from the method of the special relativity theory, when $dx^k = const$ corresponds to $dx^k{}' = const$. Now we have some new possibilities.

6.2. Approximation to the real situation

In real situation we have at least two experimental equipments and each of them has an influence at the electromagnetic field. We will take into account these influences using the 0-cohomology group $w_1(x)$ and $w_2(x)$ in accordance with the Lorentz's group dependent on 0-cohomology group. For this aim we will use two representations $Q_1^{-1} = diag(1, 1, 1, w_1)$ and $Q_2^{-1} = diag(1, 1, 1, w_2)$. We will transform the Lorentz's generators using the rule

$$\tilde{\tilde{A}}_s = Q_1^{-1} Q_2^{-1} A_s Q_2 Q_1 .$$

Following the definition

$$g = I + \tilde{\tilde{A}}_s \omega^s ,$$

we will receive the transformations [1]

$$dx' = \frac{dx - vdt}{\left(1 - w_1^2 w_2^2 \frac{v^2}{c^2}\right)^{1/2}}, \quad dt' = \frac{dt - w_1^2 w_2^2 \frac{v}{c^2} dx}{\left(1 - w_1^2 w_2^2 \frac{v^2}{c^2}\right)^{1/2}}, \; dy' = dy, \; dz' = dz .$$

The metric tensor

$$\tilde{\tilde{g}}^{ij} = diag\left(1, 1, 1, w_1^2, w_2^2\right)$$

is isometric for its action. The magnitude $\tilde{\tilde{g}}^{ij}$ describes the generalized space of the events *SE*. Now we can analyze many different situations in an unified manner. The magnitude $w_1^2 w_2^2$ governs the group. We have the Galilee's group, if $w_1 = 0$ or $w_2 = 0$. These situations realize at the initial stage of the real measurements, when the experimental equipment has no influence on the electromagnetic field corresponding $w_i \equiv 0$. We have the Lorentz's group, if $w_1 = 1$ and $w_2 = 1$. These situations realized at the final stage of the real measurements, when the influence of the experimental equipment on the electromagnetic field gives final magnitudes of the dynamical changes, corresponding $w_1 = 1$. We note, that the proposed method takes into account the different dispositions (places) of the experimental equipments, because we can use $w_1(x^i(A))$ and $w_2(x^i(B))$ where *A* and *B* are different coordinates. It is important for real experimental situation. In general case we must use the operator $A_\beta^{\beta}{}'$ in form [2]

$$\{dx^{\beta}{}'\}_{t_2, A_2, w_2} = A_\beta^{\beta}{}' \{dx^{\beta}\}_{t_1, A_1, w_1} .$$

This operator may be very complicated, following the real situation. It is clear, that this generalized kinematic method can be used additionally to the dynamical description of the electromagnetic processes.

6.3. New operation and nonassociativity

We derived some space-time transformations with different magnitudes w_i. We will introduce a new operation for the generalized Lorentz's transformations. Let

$$g_i * g_J = \pi(ij) g_i \cdot \pi(ij) g_j .$$

Here the operation $\pi(ij)$ gives

$$w(ij) = 0.5\left(w_i^2 + w_j^2\right)$$

and replaces w_i and w_j by $w(ij)$ for the elements g_i and g_j. Then we use the matrix multiplication. We will deduce for the velocities v_i, v_j that

$$v_{ij} = \frac{v_i + v_j}{1 + \frac{v_i\, v_j}{c^2} w(ij)}.$$

This law gives the nonassociativity [2], because

$$v_{ij_1k} = \frac{v_i + v_j + v_k + \frac{v_i\, v_j v_k}{c^2} w(ij)}{1 + \frac{v_i v_j}{c^2} w(ij) + \frac{w(ij, k)}{c^2} v_i\left(v_k + v_j\right)},$$

$$v_{i_1jk} = \frac{v_i + v_j + v_k + \frac{v_i\, v_j v_k}{c^2} w(jk)}{1 + \frac{v_i v_j}{c^2} w(jk) + \frac{w(i, jk)}{c^2} v_i\left(v_j + v_k\right)}.$$

The associativity corresponds to the situation, when $w_i = w_j = w_k = \ldots$ It is easy to derive the general condition for the elements (k, g, h):

$$((h * g) * g) * k = k * (g * (g * h)).$$

We have a new loop [3], for which

$$(h * k) * g \neq (h * g) * (k * g),$$
$$g * (h * k) \neq (g * h) * (g * k),$$
$$(g * h) * (g * k) \neq ((g * h) * k) * g + ((h * k) * g) * g + ((k * g) * g) * h.$$

These laws illustrate the complexity of real situations, which take place in the electrodynamics with active cohomologies, when $w(x) \neq const$. The special relativity theory corresponds to the particular case, when $w_i = w_j = w_k = 1$.

6.4. Practical aspects

1. In part 1 of this series is derived the formula for the group velocity of the electromagnetic field

$$\vec{v}_g = \frac{c}{n}\frac{\vec{K}}{K} + \left(1 - \frac{w}{n^2}\right)\left[(1 - w)\vec{u}_{fs} + w\vec{u}_m\right].$$

Here $\vec{u}_m$ is the matter velocity, $\vec{u}_{fs}$ is the primary source velocity, *n* is the rate of the refraction, *w* is the phase of the external field inertia. For $w = 0$ we have the formula

$$\vec{v}_g = c\frac{\vec{K}}{K} + \vec{u}_{fs},$$

showing the dependence of the group velocity $\vec{v}_g$ in vacuum from the primary source velocity $\vec{u}_{fs}$. But $w = 0$ for the experimental equipment corresponds to the situation without the interaction with the electromagnetic field. We must use for this case only indirect measurements.

2. In Fizo interferometer $u_{fs} = 0$ and we can use the gas, as the medium with the velocity $\vec{u}_m$. So we have the formula

$$\vec{v}_g = \frac{c}{n}\frac{\vec{K}}{K} + \left(1 - \frac{w}{n^2}\right) w \vec{u}_m .$$

We can receive the velocity $\left|v_g\right| > c$, changing the velocity $\vec{u}_m$ and the magnitude *w*.

3. Earlier we received, that the frequency of the light w_0 has the form

$$\omega = \omega_0 \frac{\left(1 - \frac{u_{fs}^2}{c^2}\right)^{1/2} - \frac{u_{fs}^2}{c^2}\Psi^{1/2}(1+\Psi)^{1/2}}{1 - \frac{u_{fs}^2}{c^2}(1+\Psi)},$$

where $\Psi = 2Q + Q^2$, $n = 1 + Q$. This law has the dependence on w and n. These results help us to make new assumptions on the behavior of the light.

Conclusions

We have shown that 0-cohomologies $w(x)$ can play the main role in the experimental investigations of the dynamic of the electromagnetic field. The Lorentz's transformations dependent on the 0-cohomology group $w(x)$ give the analytical form of the dynamical influence of the experimental devices at the electromagnetic field. By this way we receive some new physical effects.

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Part 7
NEW WAYS
FOR THE ELECTRODYNAMICS MODELS

RESUME. *It is shown, that classical electrodynamics in the spinor form has some new points for the development: non-euclidean structure of the three-dimensional space for the potential* A_n*; the possibility to use the symmetrical wave function; the classification of the topological deformations for the space of the events.*

Introduction

The present work shows the new possibilities for the development of the classical electrodynamics, which are based at its spinor form and the active 0-cohomologies for the connections between the fields and inductions. The topological sense of the metrics ξ^{ij}, which are used for the space of the events *SE*, is studied.

7.1. Non-euclidean three-dimensional space

Part 3 of this series shows the spinor form of the Maxwell's electrodynamics if we will introduce

$$\Psi = \begin{pmatrix} E_x + iB_x \\ E_y + iB_y \\ E_z + iB_z \\ 0 \end{pmatrix}, \quad \varphi = \begin{pmatrix} H_x + iD_x \\ H_y + iD_y \\ H_z + iD_z \\ 0 \end{pmatrix}, \quad \overline{\Psi} = \begin{pmatrix} E_x - iB_x \\ E_y - iB_y \\ E_z - iB_z \\ 0 \end{pmatrix}, \quad \overline{\varphi} = \begin{pmatrix} H_x - iD_x \\ H_y - iD_y \\ H_z - iD_z \\ 0 \end{pmatrix}.$$

We will find the algebraically form of this expressions using the standard connection between the fields $(\vec{E}, \vec{B})$ and the four-potentials A_k. Let [1]

$$\frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} + \frac{1}{c}\frac{\partial \varphi}{\partial t} = 0,$$

$$\Psi_1 = E_x + iB_x = -\frac{1}{c}\frac{\partial A_x}{\partial t} - \frac{\partial \varphi}{\partial x} + i\frac{\partial A_z}{\partial y} - i\frac{\partial A_y}{\partial z},$$

$$\Psi_2 = E_y + iB_y = -\frac{1}{c}\frac{\partial A_y}{\partial t} - \frac{\partial \varphi}{\partial y} + i\frac{\partial A_x}{\partial z} - i\frac{\partial A_z}{\partial x},$$

$$\Psi_3 = E_z + iB_z = -\frac{1}{c}\frac{\partial A_z}{\partial t} - \frac{\partial \varphi}{\partial z} + i\frac{\partial A_y}{\partial x} - i\frac{\partial A_x}{\partial y},$$

$$\overline{\Psi}_1 = E_x - iB_x = -\frac{1}{c}\frac{\partial A_x}{\partial t} - \frac{\partial \varphi}{\partial x} - i\frac{\partial A_z}{\partial y} + i\frac{\partial A_y}{\partial z},$$

$$\overline{\Psi}_2 = E_y - iB_y = -\frac{1}{c}\frac{\partial A_y}{\partial t} - \frac{\partial \varphi}{\partial y} - i\frac{\partial A_x}{\partial z} + i\frac{\partial A_z}{\partial x},$$

$$\overline{\Psi}_3 = E_z - iB_z = -\frac{1}{c}\frac{\partial A_z}{\partial t} - \frac{\partial \varphi}{\partial z} - i\frac{\partial A_y}{\partial x} + i\frac{\partial A_x}{\partial y}.$$

We receive for $(\partial_\tau = \partial/\partial ct)$:

$$\begin{pmatrix} -\partial_\tau & -i\partial_z & i\partial_y & -i\partial_x \\ i\partial_z & -\partial_\tau & -i\partial_x & -i\partial_y \\ -i\partial_y & i\partial_x & -\partial_\tau & -i\partial_z \\ i\partial_x & i\partial_y & i\partial_z & -\partial_\tau \end{pmatrix} \begin{pmatrix} A_x \\ A_y \\ A_z \\ -i\varphi \end{pmatrix} = \begin{pmatrix} \Psi_1 \\ \Psi_2 \\ \Psi_3 \\ 0 \end{pmatrix},$$

$$\begin{pmatrix} -\partial_\tau & i\partial_z & -i\partial_y & -i\partial_x \\ -i\partial_z & -\partial_\tau & i\partial_x & -i\partial_y \\ i\partial_y & -i\partial_x & -\partial_\tau & -i\partial_z \\ i\partial_x & i\partial_y & i\partial_z & -\partial_\tau \end{pmatrix} \begin{pmatrix} A_x \\ A_y \\ A_z \\ -i\varphi \end{pmatrix} = \begin{pmatrix} \overline{\Psi}_1 \\ \overline{\Psi}_2 \\ \overline{\Psi}_3 \\ 0 \end{pmatrix}.$$

We have, using $(a^i, b^i) \in V(4)$ (according part 3), that

$$[(-ib^1\partial_1 + ib^2\partial_2 - ib^3\partial_3) + iE\partial_T][A] = [\Psi],$$

$$[(-ia^1\partial_1 - ia^2\partial_2 + ia^3\partial_3) - iE\partial_T][A] = [\overline{\Psi}],$$

where $T = -ict$, $\partial_T = \partial/\partial(-ict)$.

We can introduce three non-euclidean metrics, which algebraically act at the four-potential A_k:

$$k^{ij} = diag(1, -1, 1, -1) = c_1 \in V(4),$$

$$e^{ij} = diag(1, -1, -1, 1) = c_2 \in V(4),$$

$$m^{ij} = diag(1, 1, -1, -1) = c_3 \in V(4).$$

The matrices (c^i, E) form the sub algebra of the $V(4)$ algebra with the conditions

$$c_i c_j + c_j c_i = \alpha_{ij}^k c_k = \{c_i, c_j\}.$$

We have also the metrics $r_{\alpha\beta}, g_{\alpha\beta}, n_{\alpha\beta}$:

$$g_{\alpha\beta} = diag(1, 1, 1, 1) = E,$$

$$r_{\alpha\beta} = diag(1, 1, 1, -1) = 0.5(E - c_1 + c_2 + c_3),$$

$$n_{\alpha\beta} = 0.5(g_{\alpha\beta} + r_{\alpha\beta}) = diag(1, 1, 1, 0).$$

We use this metrics in the spinor form of the Maxwell's electrodynamics.

We will deduce these and other metrics using

$$X = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \quad Y = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}.$$

We will receive using the tensor multiplication

$$\Pi_1 = X \otimes X = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad \Pi_2 = X \otimes Y = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix},$$

$$\Pi_3 = Y \otimes X = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad \Pi_4 = Y \otimes Y = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

So we have idempotential elements:

$$\Pi_i^2 = \Pi_i.$$

It's easy to see, that

$$E = g^{\alpha\beta} = X \otimes X + X \otimes Y + Y \otimes X + Y \otimes Y,$$

$$\left.\begin{aligned} c^1 &= X\otimes X + Y\otimes X - X\otimes Y - Y\otimes Y \\ c^2 &= X\otimes X + Y\otimes Y - X\otimes Y - Y\otimes X \\ c^3 &= X\otimes X + X\otimes Y - Y\otimes X - Y\otimes Y \end{aligned}\right\},$$

$$r^{\alpha\beta} = X\otimes X + X\otimes Y + Y\otimes X - Y\otimes Y\,,$$

$$n^{\alpha\beta} = 0.5(g^{\alpha\beta} + r^{\alpha\beta}) = X\otimes X + X\otimes Y + Y\otimes X\,.$$

We have now many metrics in Maxwell's electrodynamics with non-euclidean three dimensional spaces.

7.2. Symmetrical wave function in the electrodynamics

At the quantum level the electromagnetic field describes the quasiparticles - photons - as the particles with the spin 1. We must describe such particles, following the Pauli principle [2], by antisymmetrical wave function. The tensors (F_{mn}, H^{ik}) play this role (see part 1, part 3 of this article). We will deduce the description of the electromagnetic field in the spinor form by the symmetrical wave function, using the subgroups $(e^i, f^i) \in V(4)$ and some additional elements. Let us use the spinor's base

$$\overset{*}{\Pi}_1 = \begin{pmatrix}1\\0\\0\\0\end{pmatrix},\quad \overset{*}{\Pi}_2 = \begin{pmatrix}0\\1\\0\\0\end{pmatrix},\quad \overset{*}{\Pi}_3 = \begin{pmatrix}0\\0\\1\\0\end{pmatrix},\quad \overset{*}{\Pi}_4 = \begin{pmatrix}0\\0\\0\\1\end{pmatrix}.$$

We will construct

$$\Psi_x = E_x + iB_x\,,\quad \Psi_y = E_y + iB_y\,,\quad \Psi_z = E_z + iB_z\,,\quad \Psi_t = 0\,,$$

if we know the vectors $(\vec{E}, \vec{B})$ in the Newton's space $R^3 \times T^1$. We have now two constructions:

$$\Phi_1 = a^k\Psi_k = \begin{pmatrix} 0 & \Psi_z & -\Psi_y & -\Psi_x \\ -\Psi_z & 0 & \Psi_x & -\Psi_y \\ \Psi_y & -\Psi_x & 0 & -\Psi_z \\ \Psi_x & \Psi_y & \Psi_z & 0 \end{pmatrix},\quad e^k\Psi_k = \begin{pmatrix} 0 & \Psi_z & \Psi_y & \Psi_x \\ \Psi_z & 0 & \Psi_x & \Psi_y \\ \Psi_y & \Psi_x & 0 & \Psi_z \\ \Psi_x & \Psi_y & \Psi_z & 0 \end{pmatrix} = \Phi_2\,,$$

where $(a^k, e^k) \in V(4)$. Φ_1 is the antisymmetrical function, Φ_2 is the symmetrical function. The spinor wave function

$$\Psi = column(\Psi_x, \Psi_y, \Psi_z, 0)$$

can be received by some ways:

a) $\Psi = \dfrac{1}{4} r^{\alpha\beta} b_\alpha \left(a^k\Psi_k\right)\overset{*}{\Pi}_\beta$;

b) $\Psi = \dfrac{1}{4} g^{\alpha\beta} e_\alpha \left(e^k\Psi_k\right)\overset{*}{\Pi}_\beta$.

We have $\Psi = \dfrac{1}{8}\left\{ r^{\alpha\beta} b_\alpha \left(a^k\Psi_k\right)\overset{*}{\Pi}_\beta + g^{\alpha\beta} e_\alpha \left(e^k\Psi_k\right)\overset{*}{\Pi}_\beta \right\}$. Consequently, the symmetrical wave function is possible in the Maxwell's electrodynamics, if we use its spinor form. The permutation of the components Ψ_k gives new matrixes:

$$\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ \Psi_z \\ \Psi_y \\ \Psi_x \end{pmatrix} = \begin{pmatrix} \Psi_x \\ \Psi_y \\ \Psi_z \\ 0 \end{pmatrix}, \quad \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} \Psi_x \\ 0 \\ \Psi_z \\ \Psi_y \end{pmatrix} = \begin{pmatrix} \Psi_x \\ \Psi_y \\ \Psi_z \\ 0 \end{pmatrix},$$

$$\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} \Psi_y \\ \Psi_x \\ 0 \\ \Psi_z \end{pmatrix} = \begin{pmatrix} \Psi_x \\ \Psi_y \\ \Psi_z \\ 0 \end{pmatrix}, \quad \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \Psi_z \\ \Psi_y \\ \Psi_x \\ 0 \end{pmatrix} = \begin{pmatrix} \Psi_x \\ \Psi_y \\ \Psi_z \\ 0 \end{pmatrix}.$$

The analysis, proposed in [1], introduces the matrixes

$$\rho_i = \left\{ \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}, \begin{pmatrix} 0 & a \\ b & 0 \end{pmatrix}, \begin{pmatrix} b & 0 \\ 0 & a \end{pmatrix}, \begin{pmatrix} 0 & b \\ a & 0 \end{pmatrix} \right\},$$

which are combined from the matrixes *a* and *b*. We can consider ρ_i as the result of the new operation $*$: $\rho_i = a * b$. So we receive the coalgebra *W*(4) [3]. If we use *Q*=*a*, $b = Q * Q$, we will deduce the coassociative rule

$$Q * (Q * Q) = (Q * Q) * Q,$$

because

$$Q * (Q * Q) = \left\{ \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}, \begin{pmatrix} 0 & a \\ b & 0 \end{pmatrix}, \begin{pmatrix} b & 0 \\ 0 & a \end{pmatrix}, \begin{pmatrix} 0 & b \\ a & 0 \end{pmatrix} \right\},$$

$$(Q * Q) * Q = \left\{ \begin{pmatrix} b & 0 \\ 0 & a \end{pmatrix}, \begin{pmatrix} 0 & b \\ a & 0 \end{pmatrix}, \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}, \begin{pmatrix} 0 & a \\ b & 0 \end{pmatrix} \right\}.$$

Obviously, we have the commutativity:

$$\overset{(1)}{G} * \overset{(2)}{G} = \overset{(2)}{G} * \overset{(1)}{G}.$$

For this reason we understand, that the spinor Ψ can be combined from the symmetrical wave function Φ_2, using the algebra *V*(4) or else coalgebra *W*(4).

7.3. Four 0-cohomology dependent metrics in physical theories

The algebra $V(4)$ has two functions

$$Y_1 = Det(\lambda I - A), \quad Y_2 = Sp(\lambda I - A).$$

The elements (a^i, b^i) formed the communicative sector with $Y_1(a, b) = (\lambda^2 - 1)^2$. In the anticommutative sector for (c^i, e^i, f^i) we receive $Y_1(c, e, f) = (\lambda^2 + 1)^2$. We will present these formulas by the fig.2.

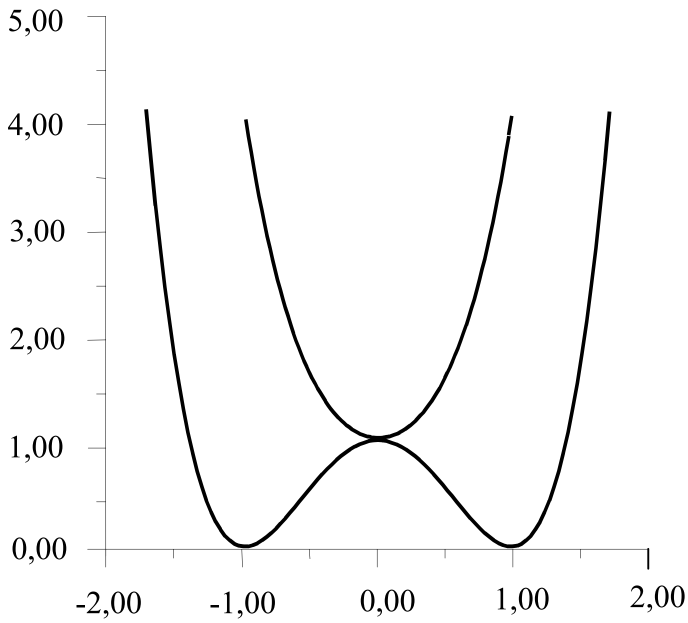

Fig.2. The 0-cogomology curves of the algebra V(4).

We can introduce 0-cohomologicaly dependent canonical metrics for the space of the events *SE* in the form

$$\xi^{ij} == diag(1, 1, 1, \lambda \cdot 1) \Rightarrow (r^{ij}, n^{ij}, g^{ij}),$$

where λ is the root of the function Y_1. We have used these metrics in the Maxwell's electrodynamics with active 0-cohomology (see part I, II, III of this article). We will study the possibilities of the metrics ξ^{ij} changing. We will introduce

$$\overset{*}{Y} = \alpha \widetilde{Y}_1 + \beta \widetilde{Y}_2$$

with (α, β) as the arbitrary functions. Let us introduce

$$\widetilde{Y}_1 = Det\left|\lambda \widetilde{I} - \widetilde{A}\right|, \quad \widetilde{Y}_2 = Sp\left|\lambda \widetilde{I} - \widetilde{A}\right|.$$

Here $\widetilde{A}$ is the matrix in which the canonical elements, equal (± 1), are replaced by arbitrary magnitudes (a, b, c, d). For example, we have the transformation:

$$A=\begin{pmatrix}0&1&0&0\\1&0&0&0\\0&0&0&1\\0&0&1&0\end{pmatrix}\Rightarrow\widetilde{A}=\begin{pmatrix}0&a&0&0\\b&0&0&0\\0&0&0&\mathrm{c}\\0&0&d&0\end{pmatrix},$$

$$I=\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix}\Rightarrow\widetilde{I}=\begin{pmatrix}a_1&0&0&0\\0&a_2&0&0\\0&0&a_3&0\\0&0&0&a_4\end{pmatrix}.$$

Thus we introduce 10-parametrical space of the 0-topological deformations with the magnitudes

$$(a;b;c;d;a_1;a_2;a_3;a_4;\alpha;\beta).$$

The function $\overset{*}{Y}$ has the form

$$V(\lambda)=\lambda^4+\sigma_2\lambda^2+\sigma_1\lambda+\sigma_0.$$

The Milnor space [3] with the coordinates $(\sigma_2,\sigma_1,\sigma_0)$ is only the three dimensional surface in the ten dimensional deformation space. The analysis of the topological deformations is known [3]. We receive all 0-cohomological dependent metrics $g^{ij}(SE)$.

The left side of each picture shows the "stability" (in the sense of the stationary phase) of the r^{ij} metrics (Euclidean's type), the right side of each picture shows the "stability" of the g^{ij} metrics (Minkowski's type). We have the system of bound states [4]. It is useful, to consider the function $\overset{*}{Y}$ as the analogy of the Higg's potential [5], supposing

$$V(\varphi)=a\varphi^4+b\varphi^2+c.$$

We understand that the "stability" of the metrics ξ^{ij} is the reaction of the electromagnetic field at the topological deformations for the space of the events *SE*. Consequently, in the classical electrodynamics there is the mechanism of the 0-cohomology dependent deformation of the field parameters, which has the analogy with the spontaneous symmetry breaking in quantum electrodynamics. This mechanism is based on the space of the events with the system of the metrics $\xi^{ij}\left(SE\right)$. The metrics ξ^{ij} are realized in the 10-parametrical space.

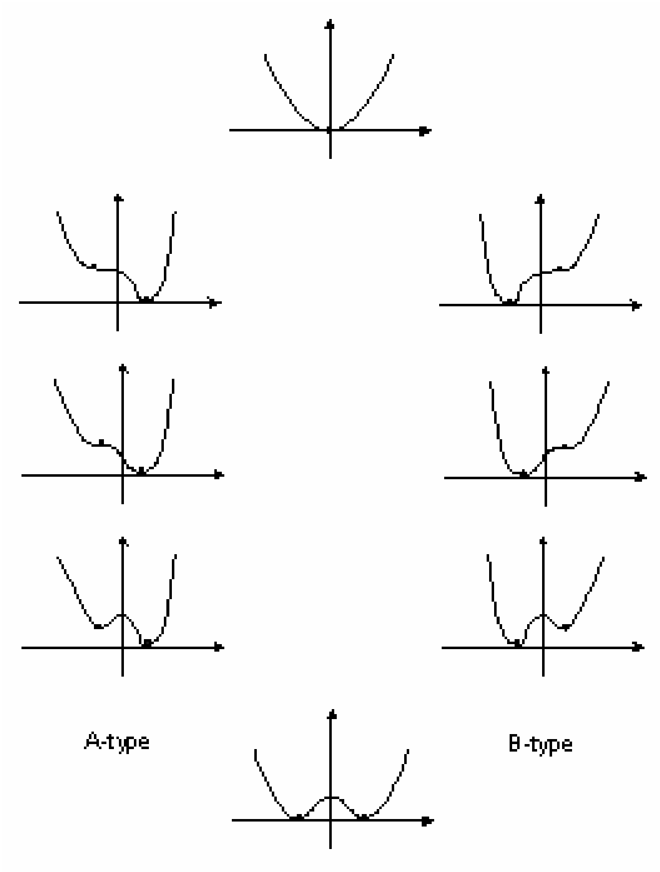

Fig.3. 0-cogomological dependent metrics ξ^{ij}

Conclusions

We have found new points for the development of the classical electrodynamics: non-euclidean structure of the three dimensional space for the potentials A_n; the possibility to use the symmetrical wave function; the classification of the topological deformations for the space of the events.

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CONCLUSIONS

Now we have dynamical model of the relativistic effects in electrodynamics. We understand that the physics of the light phenomena is based on the active 0-cohomoloqies. We are received the model without the velocity restriction and singularities. In this situation we can do new theoretical and practical steps forward to the investigation of the light particles structure.

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